Cross-Field Particle Transport due to Electromagnetic Fluctuation in a Field-Reversed Configuration

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1. Introduction

Field-Reversed Configurations (FRCs) are embedded in an external open mirror field and are sustained by the self-generated toroidal plasma current. Equilibrium of this configuration is maintained by only poloidal magnetic field. Consequently, FRCs have ultra high beta. This property is very attractive for a future reactor design because of sufficiently low input power consumed at the external coil and even in high temperature plasmas hopefully small synchrotron radiation loss; a high temperature advanced fusion such as D-^{3}He would be possible [1].

However, poorer confinement characteristic is obtained experimentally than low beta plasmas, i.e., tokamaks. Especially, the particle transport mechanism in an FRC is an ambiguous because of difficulties in diagnostic technique to measure the pulsing events. Since the obtainable data are restricted by an experimental approach, it is very important to investigate the cross-field transport in an FRC by numerical and theoretical approaches. In the history of the transport study of FRC, the lower hybrid drift instability has been believed as the dominant process [2], however, it is disproved by Carlson’s experiment [3]. The combination of radial and open-field transport is also found to be possible to modify the scaling of the gross confinement time [4]. One of the authors showed the adiabaticity breaking process near X-points [5] enhances the end loss rate, which also increases the density gradient and resultant radial flow around the separatrix [6]. Though the open-field transport is possible to affect the gross confinement time, we consider that the radial transport is still more dominant in the FRC confinement mechanisms.

Self-organized high beta plasmas fluctuate intrinsically. Fluctuating electromagnetic field in the plasmas are reported in refs. [7, 8]. In the present paper, the single particle dynamics in the electromagnetic fluctuated FRC plasma is investigated numerically. Numerical results are ensemble averaged so as to estimate the cross-field transport coefficients.

2. Model of Electromagnetic Field with Fluctuation

In order to show properties of cross-field particle transport coefficients, we observe the single particle motion inside the separatrix. The 0-th order axisymmetric FRC equilibrium is written in the form:

$$\Psi = \frac{1}{2} B_W r^2 \left(1 - \frac{r^2}{{r_s}^2} - \frac{z^2}{(l_s/2)^2} \right), \tag{1}$$

where Ψ, B_w, r_s and l_s are the flux function, the magnetic field on the midplane and separatrix, the separatrix radius, and the separatrix length, respectively. This equilibrium, i.e., Eq. (1) is known as Hill's vortex model [9]. The plasma current density of this model takes vanishing value at geometric axis and is proportional to the radial position. This profile is quite different from several experimental results that showed the hollow profiles [10, 11]. Even so, trajectories of particle motion in the configuration (1) are essentially the same as those in an FRC with the hollow current profiles. The fluctuating magnetic field is given as its divergence takes vanishing value, thus,

$$\vec{B}_1 = \nabla \times \left(\beta \vec{B}_0 \right),$$

$$\beta = \delta r_s \exp\left\{ i \left(n\theta + m\pi \frac{2z}{l_s} - \omega t \right) \right\}. \tag{2}$$

Here, the quantities δ, n, m and ω are the amplitude control parameter, the toroidal and poloidal mode number of the fluctuation, and the angular frequency, respectively. Fluctuating magnetic field $\vec{B}_1$ is automatically satisfied the condition $\nabla \cdot \vec{B}_1 = 0$. The every cylindrical component of the resultant fluctuating magnetic field is written:

$$B_{r1} = in \frac{\beta}{r} B_{z0}, \quad B_{\theta 1} = \beta \left(\frac{im\pi}{r} B_{r0} + \mu_0 j_{\theta 0} \right), \quad B_{z1} = -in \frac{\beta}{r} B_{r0}. \tag{3}$$

The subscription 0 means the equilibrium field at a steady state. Fluctuating electric field is given

$$\vec{E}_1 = i\omega\beta\vec{B}_0 . \tag{4}$$

Implicitly, Eq. (4) results from the following assumption,

$$\rho_1 = -\frac{m\pi\omega\varepsilon_0}{(l_s/2)} \beta B_z . \tag{5}$$

3. Estimation of Cross-Field Transport Coefficients by Particle Orbit Calculation and Ensemble Average

The single particle dynamics is investigated by numerical integration of the equation of motion:

$$m \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = q\left(\vec{E} + \vec{v} \times \vec{B} \right). \tag{6}$$

Here, $\vec{B}$ and $\vec{E}$ are the fluctuating electromagnetic field, which are written as

$$\vec{B} = \vec{B}_0 + \vec{B}_1,$$

$$\vec{E} = \vec{E}_0 + \vec{E}_1 = \vec{E}_1. \tag{7}$$

Without the fluctuation, the kinetic energy K=1/2(v_r^2+v_θ^2+v_z^2) is a constant of motion. In an axisymmetric FRC, the canonical angular momentum $P_\theta = mv_\theta r + q\Psi$ is also a constant of motion. When we give K and P_θ as initial conditions, the region where the particle with K and P_θ can excurse is determined. We often refer to this region as the "*accessible region*"[5], which is more popular in the space plasma physics, especially in polar aurorae, and is known as the Störmer region [12]. In the history of FRC studies, a series of Hsiao's work [13, 14] concerning with the velocity space particle loss has used the concept of this accessible region. The accessible region is defined by the following inequality equation,

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m(v_r^2 + v_\theta^2 + v_z^2) \geq U(r,z,P_\theta)$$

$$U(r,z,P_\theta) \equiv \frac{1}{2mr^2}(P_\theta - q\psi(r,z))^2. \tag{8}$$

Here, $U(r,z,P_\theta)$ is the effective potential known as the Störmer potential [12]. The inequality contains the space variables as well as two constants of motion, K and P_θ. Thus location of the accessible region is determined by giving the values of K and P_θ of particles concerned. Although there are some exceptions, the value of P_θ characterizes the guiding center surface and K gives the width of the accessible region. Since we define $\Psi \geq 0$ inside the separatrix, the bulk plasma ions (ions inside the separatrix) have the positive value of P_θ. A detailed calculation shows us, however, for an ion with positive P_θ an existence of the accessible region that extends over open-field region.

To estimate the transport coefficients, statistical properties are investigated according to ref. [15]. As we discussed above, the canonical angular momentum is observed for estimation of the cross-field transport coefficients. The displacement and ensemble average of the physical quantity X among N particles are calculated by

$$\delta X(t) = X(t) - X(0), \quad \langle X(t) \rangle = \frac{1}{N}\sum_{i=1}^{N} X_i(t). \tag{9}$$

The first and second cumulant are written in the following form:

$$C_1(t) = \langle \delta X(t) \rangle, \quad C_2(t) = \left\langle \left(\delta X(t) - \langle \delta X(t) \rangle\right)^2 \right\rangle. \tag{10}$$

The first cumulant of P_θ describes the guiding center drift, and the second one shows the cross-field particle diffusion. Being used the second cumulant of P_θ, the cross-field diffusion coefficient is calculated by

$$D_{P\theta} \equiv \lim_{t\to\infty} \frac{C_{2P\theta}(t)}{2t}. \tag{11}$$

Even if the diffusion coefficient is finite, no particle flow takes place in uniform plasmas. In

FRCs, however, steep density gradient is observed near the separatrix. The combination of the finite diffusion coefficient and the density gradient causes the cross-field particle flow; this gives a deleterious effect on the particle confinement.

4. Results and Discussion

We traced numerically the particle orbit in the fluctuating electromagnetic field of FRC. The time evolutions of the first cumulant of P_θ, i.e., $C_{1p\theta}(t)$ are calculated for 1000 particles with initial P_θ of 0.01. The toroidal mode numbers n are 1, 2 and 3, and the poloidal mode number m is 1. It is found that the first cumulants oscillate together with the fluctuation, and deviate from the initial value of 0.01. However, even the maximum deviation at t = 4000 is within 0.3%, thus the time derivative of guiding center shift appears to be negligibly small. The large amplitude of oscillation is observed for larger toroidal mode due to severer breaking of axisymmetry. The quantity $C_{2p\theta}(t)/2t$ is calculated to find the cross-field diffusion coefficient. As shown in Fig. 1, the quantities $C_{2p\theta}(t)/2t$ for the toroidal mode numbers n of 1, 2 and 3 approaches certain constant value. According to Eq. (6), this constant value is the cross-field diffusion coefficient $D_{p\theta}$. It is found that the fluctuation with the larger toroidal mode diffuses particles deleteriously because of the larger cross-field diffusion coefficient.

The toroidal mode number dependence of diffusion coefficient is shown in Fig. 2. The poloidal mode number is fixed to be 1 here. The solid curve is drawn by the relation $D_{P\theta} \propto n^2$. The frequency dependence of diffusion coefficient is arranged in Fig. 3. The solid curve is drawn by $D_{P\theta} \propto (\omega / \omega_c)^{-2.2}$, where ω_c is the ion cyclotron frequency on the separatrix and midplane. We found fluctuations of lower frequency cause a deleterious effect on the cross-field particle transport. Finally, the dependence of the diffusion coefficients on the kinetic energy K is presented in Fig. 4. In this figure, the kinetic energy K is transformed to the stability parameter S by the form $S \equiv (r_s - R)/\rho_i$, where $\rho_i = \sqrt{2mK}/(qB_w)$. Although the definition of the conventional stability parameter is $\bar{s} \equiv \int_R^{r_s} r dr/(r_s \rho_{Li})$ and is different from the above S, typical characteristics of the diffusion process can be described also by S. In the form of $\bar{s}$, ρ_{Li} is the local ion gyroradius. Higher energy ions diffuse out faster from the separatrix due to the large diffusion coefficient than lower energy ions.

We can estimate roughly the particle confinement time by using $\tau_N \propto \Psi_{\max}^2 / D_\Psi$, where $\Psi_{\max}$ is flux function at the magnetic axis. The diffusion coefficient we calculated $D_{P\theta}$ is easily converted to D_Ψ by using $D_{P\theta} \propto q^2 D_\Psi$. The solid curve in Fig. 4

represents $D_{P\theta} \propto S^{-3.156}$, thus the scaling of τ_N with respect to S becomes $\tau_N \propto S^{3.156}$. About two times larger power with respect to S is found than the one experimentally obtained; the scaling analyzed with several experimental results is reviewed in [16].

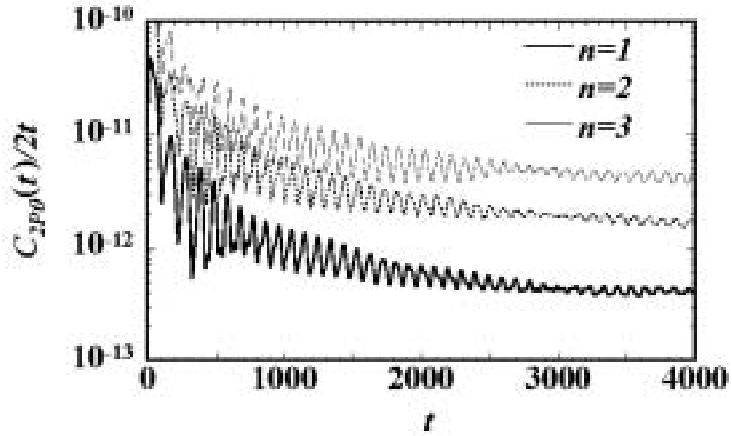

Fig. 1: The time evolutions of the second cumulant of P_θ divided by 2t, i.e., $C_{2p\theta}(t)/2t$. The thick solid line is for n=1 fluctuation, the dotted line is for n=2 and the thin solid line is for n=3.

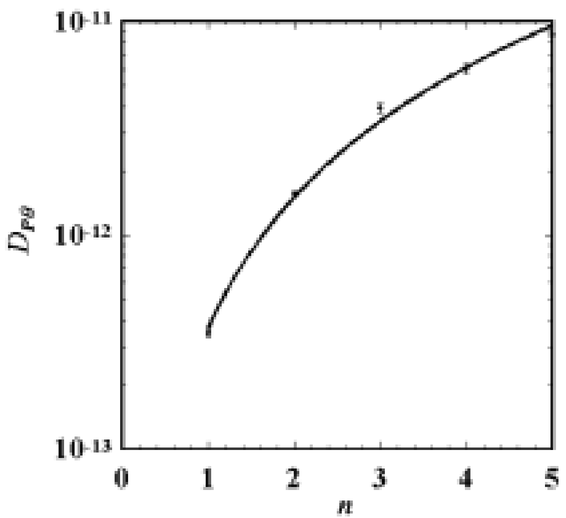

Fig.2: The dependence of diffusion coefficient on the toroidal mode numbers n. The solid curve is drawn by the following relation: $D_{P\theta} \propto n^2$.

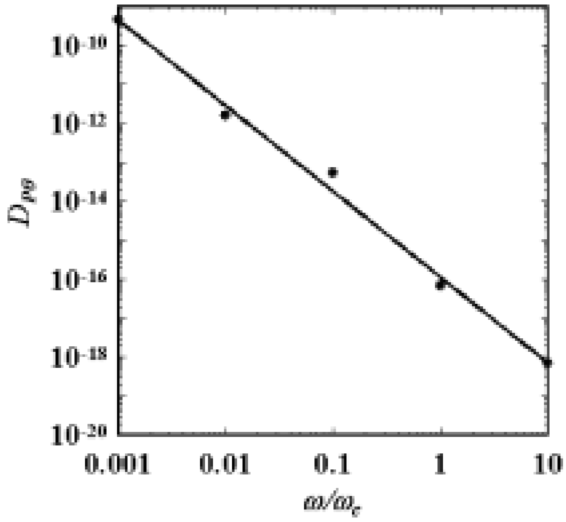

Fig. 3: The dependence of the diffusion coefficient on the fluctuation frequency. The solid curve is drawn by $D_{P\theta} \propto (\omega/\omega_c)^{-2.2}$.

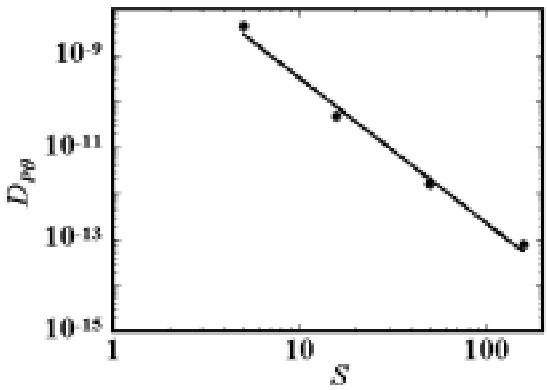

Fig. 4: The dependence of the diffusion coefficient of the canonical angular momentum on the kinetic energy K. The kinetic energy K is transformed to S in the form $S \equiv (r_s - R)/\rho_i$, *where* $\rho_\mathrm{i} \equiv \sqrt{2mK}/(qB_\mathrm{w})$.

5. Conclusions

Investigation on the cross-field particle transport in a Field-Reversed Configuration (FRC) due to the electromagnetic fluctuation has been carried out. The fluctuation is described by the toroidal and poloidal mode numbers. We have traced numerically various orbits of many ions and observed the temporal change of the canonical angular momentum for the ions in the fluctuating field; this quantity is a constant of motion in axisymmetric and non-fluctuating FRC and is a good measure for the location of the guiding center. In order to estimate the cross-field transport coefficients, the first and second cumulants of the canonical angular momentum have been calculated. From the evolutions of the first cumulant of the canonical angular momentum, it is found that the cross-field drift of plasma ion can be neglected safely. The fluctuation, however, causes the diffusive flow near the separatrix, where the density gradient is large. The dependences of the diffusion coefficients on the various parameters have been studied. We have examined the fluctuation effects in the lower frequency range than ion cyclotron frequency. It is found that the fluctuation with higher toroidal mode and lower frequency affect the cross-field particle diffusion deleteriously. The scaling of the particle confinement time has been estimated with respect to the stability parameter *S*, and is found to be proportional to $S^{3.156}$. It appears that about two times stronger dependence on *S* is obtained compared with the experimental results.

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