SELF-CONSISTENT ELECTRIC FIELDS AND ROTATION IN FRCs

Loren Steinhauer University of Washington

Abstract

Shorting of open field lines where they intersect external boundaries strongly modifies the electric field all along the field lines.[1] However, it doesn't simply force the radial electric field to zero. The self-consistent electric field is found by an extension of the familiar Boltzmann relation for the electrical potential. The resulting electric field points *outward*. The resulting rotational flow has effects in three important areas: (1) the plasma rotation rate; (2) the particle-loss spin-up mechanism; and (3) the sustainability of rotating magnetic field current drive.

I. Introduction

Field-reversed configurations (FRC) still conceal many mysteries concerning stability, transport, the presence of flow, and edge-layer anomalies. Here the focus is on the edge-layer. The *end flow* of plasma from the edge layer is well known to be anomalously slow, and no adequate physical explanation for this has yet been established [1]. The question of *rotational flow* in the edge layer is taken up, as has been analyzed recently [2]. The particular effect investigated here is end-shorting, which can take place on open fields lines. End shorting modifies the relative electric potential on adjacent magnetic field lines. The self-consistent electric potential can be found by an extension of the familiar Boltzmann relation. The surprising result is that the self-consistent radial electric field actually points outward. This is contrary to the conventional rule of thumb that end-shorting forces the radial electric field to zero. The radial electric field, determined in this way, controls the rotational motion of the plasma. This has a profound effect on several issues related to the plasma rotation, as will be seen.

The convention adopted here for the magnetic field is shown in Fig. 1



Fig. 1. Conventions and rotation in FRCs

II. Boltzmann relation

The plasma rotation is determined by a combination of the Boltzmann relation for the electrical potential, and the ion fluid drifts. The Boltzmann relation follows from the parallel component of the electron equation of motion

$$\phi = (kT_e/e)\ln n + \phi_0(\psi) \tag{1}$$

Here ϕ is the electrostatic potential, ϕ_0 is an integration "constant" which actually depends on the magnetic surface (variable ψ) since the integration is along a field line. In the unshorted case the potential ϕ floats freely on each magnetic surface $\psi = const$. In the shorted case, $\phi_0(\psi)$ uniform from surface to surface. In this case the radial electric field $E_r = -\partial \phi / \partial r$ is *outward* because $\partial n / \partial r < 0$.

Consider now the fluid drifts. In an elongated FRC the fluid drift (azimuthal) for the species $\alpha = i, e$ is

$$u_{D\alpha} = \frac{c}{B} \frac{d\phi}{dr} + \frac{c}{q_{\alpha} n B} \frac{dp_{\alpha}}{dr}$$
(2)

The first term is the electric drift V_E and the second is the diamagnetic drift $v_{d\alpha}$. In the case of a non-rotating ion fluid, the electric drift exactly cancels the ion diamagnetic drift. Note that v_{di} is negative (since $dp_i/dr < 0$ and *B* is positive at the edge). Thus in the non-rotating case the electric drive must be positive. Since $E_r = -d\phi/dr$, the electric field must be inward in the nonrotating case.

These drifts can be characterized using familiar parameters. The eta parameter is compares the temperature and density gradients for each species: $\eta_{\alpha} \equiv |\nabla \ln T_{\alpha}| / \nabla \ln n$. The density gradient length scale is $1/L_n \equiv |\nabla n/n|$. Beta for each species is based on the local magnetic field are $\beta_{\alpha} \equiv 8\pi nkT_{\alpha}/B^2$. Then the diamagnetic and electric drifts are

$$V_{di} = -\frac{(1+\eta_i)\beta_i}{2} \frac{V_A \ell_i}{L_n}; \qquad V_E = -\frac{(1+\eta_e)\beta_e}{2} \frac{V_A \ell_i}{L_n}$$
(3)

where ℓ_i and V_A are the local ion skin depth and Alfven speed, respectively. Note the similarity of form for the two drift expressions. The ion diamagnetic drift is *negative*, i.e. in the $-\theta$ direction. This is the same as the direction of ion gyromotion (see Fig. 1). The electric drift here is for the end-shorted case, i.e. we have used the Boltzmann equation (1) with $d\phi/d\psi = 0$. Note that in the shorted case the electric drift is also *negative*. Thus end-shorting causes the ion fluid to rotate *even faster* than the ion diamagnetic drift.

Consider reasonable values of the parameters. Based on very limited observations in FRCs and reasonable guesses, $\eta_e = 2/3$ and $\eta_i = 1/3$. A typical value of the beta based on the external field (B_{ext}) at the separatrix is 0.6. This corresponds to $\beta_i = 1$ and β_e for the typical case $T_i/T_e = 2$. For these values the two drifts are

$$V_{di} = -\frac{2}{3} \frac{V_A \ell_i}{L_n}; \qquad V_E = -\frac{5}{12} \frac{V_A \ell_i}{L_n}$$
(4)

III. Plasma rotation rate

Consider now three applications of theory of plasma rotation in the end-shorted case. The first is the magnitude and structure of the rotation itself. The familiar parameter for measuring the rotation rate is $\alpha \equiv \Omega_i / \Omega_{di}$ where $\Omega_i = u_{i\theta}/r$ is the ion fluid rotation frequency, and $\Omega_{di} = u_{di\theta}/r$ is the ion diamagnetic drift frequency.

Few measurements have been made of plasma rotation in ordinary FRCs (not driven by rotating magnetic fields). Where it has been measured using the Doppler shift of a carbon-V line [3], the inferred value of the rotation parameter is in the range $\langle \alpha \rangle \sim 0.4 - 1$. This is the average or "bulk" value $\langle ... \rangle$ of the rotation since the experiments did not attempt to resolve the structure of the rotation.

Using the end-shorting theory, the value of α in the edge can be determined. Using data from the FRC data compendium [4] in instances where the density gradient length in the edge can be inferred [5], the edge values of α are as shown in Tab. I.

		-
device	log#	α_{edge}
FRX-B	1	3.2
FRX-C	2	4.6
FRX-C	1	4.7
TRX-1	2	5.3
TRX-1	3	5.6
TRX-1	1	8.6
TRX-2	9	5.0

Tab. I. Euge Iolation III I'KC	Tab. I	. Edge	rotation	in	FRC
--------------------------------	--------	--------	----------	----	-----

Here "log#" refers to the log number in the compendium. The edge values are in the range $\alpha \sim 3 - 8$. This is several times larger than the bulk values $\langle \alpha \rangle \sim 0.4 - 1$. This is evidence for strongly sheared-rotational flow in FRCs. Not only do FRCs rotate, but the rotation rate is much faster in the edge than in the core.

IV. Spinup mechanism

The second application is to the *particle-loss spinup* mechanism proposed originally by Eberhagen and Grossmann [6]. This is one of two spinup mechanisms, the other being end-shorting induced rotation, which is transmitted to the plasma by ion viscosity [7]. The *particle-loss spinup* mechanism is based on the following hypothesis. End-loss of ions produces a rocket-like rotation effect on the plasma. If the typical ion that is lost out the ends (presumably originating from near the plasma edge) had been rotating in the <u>plus- θ direction</u>, then the plasma loses a positive quantum of momentum. Thus the remaining plasma experiences a recoil in the *minus-\theta* direction. The grad-*B* drift (a particle drift) is indeed in the *plus-\theta* direction (as will be seen shortly), and the minus θ direction is the normal direction of spinup. Thus *particle-loss spinup* is a plausible mechanism to explain the spinup of FRCs.

The grad-B drift for an ion with the transverse "thermal" velocity $v_{thi} = (kT_i/m_i)^{1/2}$ in an elongated FRC is

$$V_{\nabla Bi} = \frac{v_{thi}^2}{2\omega_{ci}} \frac{1}{B} \frac{dB}{dr}$$
(5)

where $\omega_{ci} = eB/m_ic$ is the ion cyclotron frequency. Assuming pressure balance $p + B^2/8\pi = const$, and the parameters defined earlier, the grad-B drift can be written in the form

$$V_{\nabla Bi} = \frac{\beta_i}{8} [(1+\eta_e)\beta_e + (1+\eta_i)\beta_i] \frac{V_A \ell_i}{L_n}$$
(6)

For the typical parameters used earlier this simplifies to

$$V_{\nabla Bi} \approx + \frac{13}{48} \frac{V_A \ell_i}{L_n} \tag{7}$$

The particle-loss spinup hypotheses as it has been presented elsewhere is incomplete in that only the grad-B drift was accounted for. In effect it was assumed that shorting would force the radial electric field to zero. As shown here (Eqs. 3b,4b), E_r is *not* zero in the shorted case, and thus will also contribute to the particle drift. If the ions that are lost originate from the plasma edge (as is expected), then the electric field should be the end-shorted field which is not zero. Compare Eqs. (3b,7): the electric drift *opposes* the grad-B drift and exceeds it in magnitude by about 50%. The upshot is that the particle-loss spinup carries away a *negative* momentum, and would cause the plasma to spin up in the + θ -direction. This is contrary to the consistent observation in FRCs that the plasma spins up in the $-\theta$ -direction [8].

V. Ion spinup in RMF

Rotating magnetic field (RMF) current drive is an attractive method for current drive in FRCs. The rotating field with frequency ω_{RMF} exerts a forward torque on the electrons so long as ω_{RMF} exceeds the electron rotation frequency Ω_e . This torque can balance the opposing torque of the electrons dragging against the ions. However, a liability of RMF is that it only directly drives the electron current. Sustainable RMF requires that the ions *not* spin enough appreciably. If, however, the ions are "free-wheeling," i.e. there is no torque on the ions except friction against the electrons, then RMF current drive cannot be sustained. The reason for this is tied up with the electric fields and resulting electric drift.

If the ions are free-wheeling, then friction from the rotating electrons will cause the ion fluid to spin up. This, in turn, modifies the radial electric field, as seen by rearranging Eq. (2),

$$\frac{cE_r}{rB} = -\Omega_i + \frac{c}{enB}\frac{dp_i}{rdr}$$
(8)

spinning up the ions ($\Delta\Omega_i > 0$) causes a negative change in the radial electric field ($\Delta E_r < 0$). This causes a positive change in the electron rotation ($\Delta\Omega_e > 0$) that is exactly the same as $\Delta\Omega_i$. As a result the electron rotation Ω_e catches up with the ω_{RMF} . The result is the loss of RMF torque on the electrons, turning off the electron current drive effect. At this point the current begins to decay as if there were no current drive at all. The transient behavior of RMF was addressed in greater detail elsewhere [9].

Sustainable RMF current drive, however, is possible if there is an independent torque on the ions. This can be provided by drag against a background population of neutral gas, or by neutral beam injection. A third possible source, introduced here, is the effect of end-shorting. End-shorting reverses the causality. In the case of freely-floating potential the causality is $\Omega_i \rightarrow \phi$ and E_r . With end-shorting, the causality is "shorting" $\rightarrow \phi \rightarrow \Omega_i$. This imposes an independent torque on the ions if shorting takes place where the field line has its "footprint" on the boundary. Then, by line tying the field lines can twist so as to exert a torque on the ion fluid.

Ordinarily, shorting is something that occurs on open field lines, namely outside the separatrix. However in the case of RMF the transverse field opens up the field lines inside the nominal separatrix so that even the interior of the FRC can be shorted. This is illustrated in Fig. 2. Thus end-shorting offers the possibility of an independent torque on the ions so that RMF current drive can be sustained.



Fig. 2. Open field line penetration into the FRC interior by RMF.

Two key questions remain however. (1) Can end-shorting be assured in spite of the fact that the "footprint" where the field lines that touch the boundary? Since the footprint moves in a cyclic pattern at the boundary, it may not be line-tied in the traditional sense. A metal "limiter" at the boundary contact point may be needed to assist the end-shorting. (2) The second question concerns electron energy loss: won't opening up the "interior" field lines cause a disastrously fast electron thermal loss? This is a critical issue wholly apart from the current-drive sustainability.

The electron thermal loss issue was taken up elsewhere [10]. An analysis of experimental results showed that the energy lost per electron flowing out the end is $W_e \sim (4-8) kT_e$. This relationship holds for a wide range of collisionalities achieved in experiments, $10^{-3} < \lambda_e/L_{conn} < 2$, where λ_e is the electron mean-free-path, and L_{conn} is the

connection length to end wall. Thus the electron thermal loss is *convective* which is much less than conductive, especially at high temperature fusion conditions.

VI. Summary

End shorting of open field lines determines the electrical potential and radial electric field which in turn determine the ion rotation. There are at least three important consequences of this. (1) The ions in the edge layer rotate quite rapidly. Comparison with experimental inferences of the rotation in the interior shows that the edge plasma is rotating at a much higher rate than the interior; i.e. the rotational flow is strongly sheared. (2) The self-consisted electric field produces an electric drift of the ions in the edge of the plasma that opposes the grad-B drift. Moreover, the electric drift is larger in magnitude. Thus the particle-loss spinup hypothesis should be discarded because it predicts spinup in the wrong direction. (3) End-shorting offers an independent source of torque on ions; this may allow RMF, which needs an additional torque on the ions, to be sustainable.

References

1. L.C. Steinhauer, Phys. Fluids 29, 3379 (1986).

2. L.C. Steinhauer, Phys. Plasmas 9, September 2002 issue (in press)

3. W.T. Armstrong, R.K. Linford, J. Lipson, D.A. Platts, and E.G. Sherwood, Phys. Fluids **24**, 2068 (1981).

4. Appendix of N. Iwasawa, A. Ishida, and L.C. Steinhauer, Phys. Plasmas 7, 931 (2000).

5. L.C. Steinhauer, Phys. Fluids 29, 3379 (1986).

6. A. Eberhagen and W. Grossmann, Z. Phys. 248, 139 (1971);

7. L.C. Steinhauer, Phys. Fluids 24, 328 (1981).

8. While few direction measurements of the rotation have been made, many observations show that the rotational instability mode rotates in the $-\theta$ -direction. According to the stability theory the plasma rotation and mode rotation are in the same direction.

9. L.C. Steinhauer, Phys. Plasmas 8, 3367 (2001).

10. L.C. Steinhauer, Phys. Fluids B 4, 4012 (1992).