Self-generation of Hollow Current Profile and Tilt Instability in a Field-Reversed Configuration

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Abstract. The profile relaxation from a magnetichydrodynamic (MHD) profile to a kinetic equilibrium in field-reversed configurations (FRCs) is investigated by two-dimensional electromagnetic particle simulation. The radial oscillation takes place in order to relax an excess energy in the MHD profile, and the system spontaneously relaxes toward a kinetic equilibrium. In this kinetic equilibrium, the hollow electron current profile is realized as a result of the combined effects of the single particle orbits and the ion finite Larmor radius, and the ion current profile becomes peaked due to the effect of the ion meandering motion. Three-dimensional full electromagnetic particle simulation is also performed to study the stability of these kinetic equilibrium against the tilt mode. The growth rate of the tilt instability is reduced by the kinetic effects. It is found that the stabilization effect of tilt mode becomes much distinct when the current density changes from the peaked profile to the hollow one.

1. Introduction

The tilt instability in the field-reversed configuration (FRC) plasma is predicted by the magnetichydrodynamic (MHD) theory[1,2], but it has not been observed in the experiments[3]. It is also reported that most experimental equilibrium tend to take a hollow current profile[4]. This tilt instability has been studied by the extended MHD models, but they could not give the satisfactory explanation as yet[5-16]. An MHD equilibrium was used as the initial condition for the three-dimensional (3D) electromagnetic particle simulation[12-14]. However, the influence of the kinetic effect on the tilt mode was not clarified, because the MHD equilibrium relaxes to the kinetic one simultaneously with the evolution of the tilt instability.

It is important to obtain a kinetic profile of an FRC plasma and examine its character in considering the tilt instability, because most experimental FRC plasmas are kinetic. Though it is quite difficult to give an analytic solution of a kinetic FRC equilibrium, we can clarify what kind of an equilibrium is realized in a kinetic FRC plasma separately from the evolution of the tilt mode by perfoming two-dimensional (2D) electromagnetic particle simulation. Because the tilt mode is 3D instability. This 2D simulation is first carried out to sufficiently relax the system from an initial MHD profile to a kinetic equilibrium. In Sec. 3, the relaxation process of an FRC plasma from an MHD profile to a kinetic equilibrium and the physical property of the kinetic equilibrium are studied. And then we perform 3D full electromagnetic particle simulation in which the kinetic equilibrium obtained from the 2D simulation is used as the initial condition. Section 4 is devoted to discussing how the kinetic FRC plasma is kept stable aginst the tilt instability. Summary is given in Sec.5.

2. Simulation Method

Taking it into account that the tilt mode is 3D instability, we first perform 2D electromagnetic particle simulation to get the kinetic equilibrium without exciting the tilt instability, and clarify the property of the kinetic FRC plasma. An initial profile for 2D simulation is given by a one-fluid MHD equilibrium which is controlled by the hollowness parameter D, the plasma beta value $\beta_{\rm sp}$ at separatrix and the finite Larmor radius (FLR) parameter $\overline{s}$[11]. For simplicity, we assume that initial particle distribution is the shifted-Maxwellian under the zero $E(0)$ condition, where the average flow velocity is equal to the diamagnetic velocity. In this distribution, the ion temperature and the electron temperature are the same and spatially constant. In this initial condition, it is natural choice in order to examine the generation of E in the simulation. Next we examine the feature of the tilt instability by means of 3D full electromagnetic particle simulation in which the kinetic equilibrium obtained from 2D simulation is used as the initial condition. Both 2D and 3D code relies on the semi-implicit method[17].

It is assumed that the physical quantities are periodic at the boundary of the z-axis ($z = \pm z_D$) and the cylindrical vessel wall ($r = r_D$) is a rigid perfect conductor at which particles are elastically reflected. The half-height of this vessel z_D is fixed to three times the vessel radius r_D in this paper ($z_D = 3r_D$). The total number of particles is 10^6 in this simulation. The ratio of ion to electron mass m_i/m_e is 50 and the frequency ratio ω_{pe}/ω_{ce} is 5, where $\omega_{pe}(= \sqrt{\frac{4\pi n_e q_e^2}{m_e}})$ is the electron plasma frequency defined by the density n_e at the field-null line and $\omega_{ce}(= \frac{q_e B_{wall}}{m_e c})$ is the electron cyclotron frequency defined by the magnetic field B_{wall} at the vessel wall in the equatorial plane ($z = 0$). We adopt the predictor-corrector method with a sub-stepping of the electromagnetic field for a time advancing. Namely, the particles are advanced with a large time step of $\omega_{pe}\Delta t = 1.5$, where Δt is a time step[17], while the electromagnetic field is advanced with a small time step of $(c\Delta t)/(\Delta\phi\Delta r) < 1.0$, where $\Delta\phi\Delta r$ is a minimum grid separation[18], by using an iteration method.

The parameter $\beta_{\rm sp}$ is fixed to 0.2 because the growth rate of the tilt instability is the smallest at this value in the simulations done by Nishimura *et al.*[12] On the other hand, we adopt two types of current profile models, i.e., a peaked profile ($D = -0.6$) and hollow profile ($D = 0.2$), as a initial value for the MHD equilibrium in order to investigate the dependence of the kinetic equilibrium on the initial current profile and to examine whether the hollow profile is spontaneously realized in an FRC plasma.

We furthermore adopt the parameter $\overline{s}$ for the ion FLR effect as the another control parameter. The parameter $\overline{s}$ is defined by

$$\overline{s} = \int_R^{r_{\rm sp}} \frac{r dr}{r_{\rm sp}\lambda_i}, \tag{1}$$

where $r_{\rm sp}$ is the separatrix radius, R is the radius of the field-null line, and λ_i is the local ion gyroradius at $z = 0$. This parameter roughly indicates the ratio of the plasma radius to the ion Larmor radius. In this paper, we examine what kind of kinetic profile is realized and how the feature of the tilt mode is altered, when $\overline{s}$ varies from a full kinetic case ($\overline{s} = 1$) to a moderate kinetic case ($\overline{s} = 3$).

3. Profile relaxation

Let us examine how the plasma profile changes from an MHD profile to a kinetic equilibrium based on 2D simulation results. Figure 1(a) and 1(b) show the time evolutions of the field-null line and separatrix radii (R and $r_{\rm sp}$) on the midplane in (a)$\overline{s} = 1$ and (b)

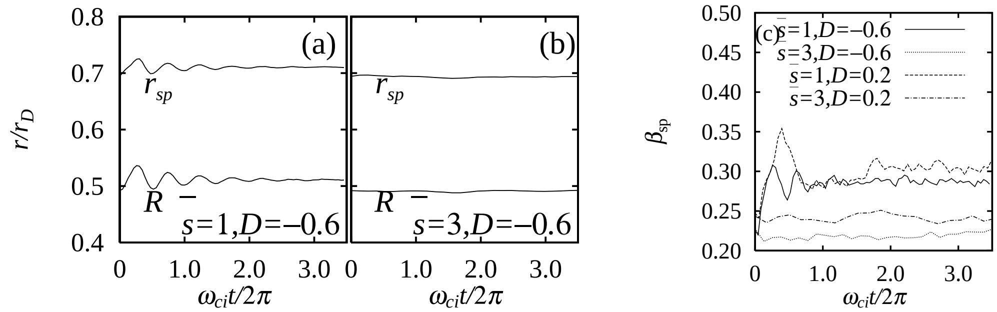

FIG. 1: *Time evolution of the field-null line and separatrix radii (R and $r_{\rm sp}$) on the midplane in (a)$\overline{s} = 1$, (b)$\overline{s} = 3$, and (c)time evolution of the plasma beta value at the separatrix ($\beta_{\rm sp}$). The solid line shows the case of $\overline{s} = 1$ and $D = -0.6$, the dotted line shows the case of $\overline{s} = 3$ and $D = -0.6$, the broken shows the case of $\overline{s} = 1$ and $D = 0.2$, and the dashed-and-dotted line shows the case of $\overline{s} = 3$ and $D = 0.2$, respectively.*

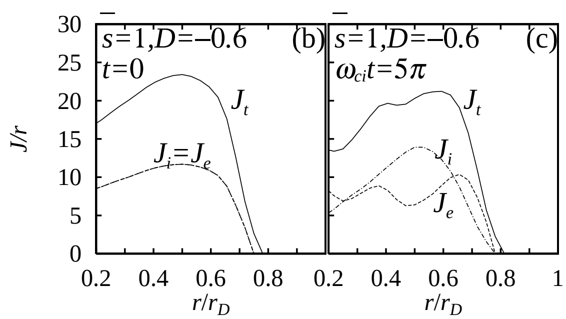

FIG. 2: *The radial profile of toroidal current density on the midplane in $\overline{s}$=1 at (a)$\omega_{ci}t$=0 and (b)$\omega_{ci}t = 5\pi$. The solid line shows the total current density, the broken line shows the electron current density, and the dashed-and-dotted line shows the ion current density, respectively.*

$\overline{s} = 3$, respectively.

In the full kinetic case ($\overline{s} = 1$), both R and $r_{\rm sp}$ oscillate with frequency $\omega \sim 2\omega_{ci}$ in the early period and damp gradually until $\omega_{ci}t \sim 5\pi$ shown in Fig. 1(a). The plasma beta value $\beta_{\rm sp}$ at the separatrix jumps from an initial small value to about 0.3 in an initial moment, and keeps this value after that (Fig. 1(c)). This phenomena indicates that the profile oscillates in the radial direction to relax an excess energy in an MHD profile. In the moderate kinetic case ($\overline{s} = 3$), on the other hand, no oscillation appears (Fig. 1(b)) and $\beta_{\rm sp}$ keeps an initial value (Fig. 1(c)). When a plasma is fully kinetic, the energy difference between an initial MHD profile and an obtained kinetic equilibrium is so large that a relaxation oscillation is excited.

Figure 2 shows the radial profiles of toroidal current density on the midplane in $\overline{s} = 1$ at (a)$\omega_{ci}t = 0$ and (b)$\omega_{ci}t = 5\pi$, respectively. After the relaxation oscillation, the electron current density J_e increases near the separatrix, and decreases near the field-null line. An initial peaked profile ($D < 0$) changes to a hollow profile ($\tilde{D}_e > 0$). On the other hand, the ion current density J_i becomes more peaked ($\tilde{D}_i < D < 0$). So the total current J_t changes to the hollow profile near the field-null line. Both the decrease of J_e and the increase of

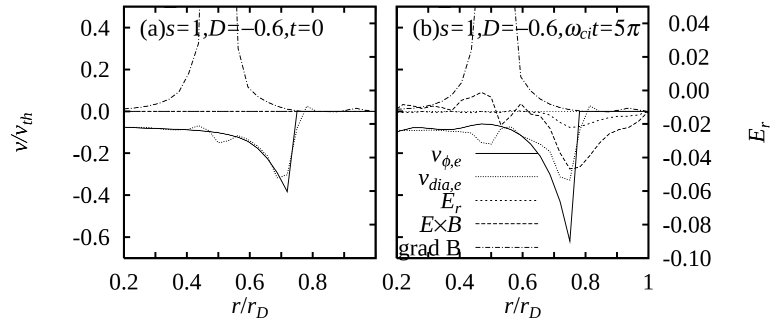

FIG. 3: *The radial profile of toroidal electron flow velocity on the midplane in $\overline{s}$=1 at (a)$\omega_{ci}t$=0 and (b)$\omega_{ci}t = 5\pi$. The solid line shows the electron velocity, the dotted line shows the electron diamagnetic velocity, the dashed line shows the radial electric field, the broken line shows the $E \times B$ drift, and the dashed-and-broken line shows the gradient-B drift, respectively.*

J_i near the field-null line can be explained by the character of the single particle orbit (Fig. 3). The dominant electron motion near the field-null line is the gradient-B drift. Because the gradient-B drift has the opposite sign to the electron diamagnetic drift, J_e decreases near the field-null line. On the other hand, when the spatial scale of magnetic field is almost the same as the ion orbit scale, ions execute meandering motions along the field-null line. The average toroidal velocity is so large due to this meandering motion that J_i increases near the field-null line. Next, we consider why J_e increases near the separatrix (Fig. 3). Since the density profile becomes steep locally in the narrow periphery region near the separatrix, the ion FLR effect generates the strong radial electric field E_r there. Because the generated $E \times B$ drift has the same sign as the electron diamagnetic drift, J_e increases in the periphery. On the other hand, E_r acts on ions less effectively since the ion Larmor radius is larger than the spatial size of a strong electric field region. That is, the modification of ion current profile becomes relatively smaller.

In this way, an initial MHD equilibrium with the peaked current profile relaxes to a kinetic equilibrium with the hollow current profile through the effects of the single particle orbit and FLR. We find the tendency for the electron current to become a hollow profile and for the ion current to become a peaked profile independently of the initial condition, such as the initial hollowness parameter D and the FLR effect $\overline{s}$.

4. Tilt instability

We clarify from 2D simulation that the kinetic equilibrium with the hollow current profile is spontaneously generated in the FRC plasma. In this section, the feature of the tilt mode in the kinetic equilibrium is also investigated based on 3D simulation results. In 3D simulation, the kinetic equilibrium solution obtained after the profile relaxation in 2D simulation is adopted as the initial condition.

There are several parameters which are related to the stabilization of the tilt mode. That is, the plasma beta value β, the hollowness parameter D, the Alfvén Mach number M_A, and the FLR parameter $\overline{s}$. We discuss the relationship between the growth rate of the tilt mode and these parameters. Figure 4 shows the dependence of the tilt growth rate γ_{tilt} on (a) the plasma beta value $\beta_{\rm sp}$ at the separatrix, (b) the electron hollowness parameter $\tilde{D}_e$ and (c) the Alfvén Mach number M_A, where γ_{tilt} is normalized by those obtained from MHD simulation γ_{MHD}, and M_A is the associated with the ion toroidal flow velocity, the

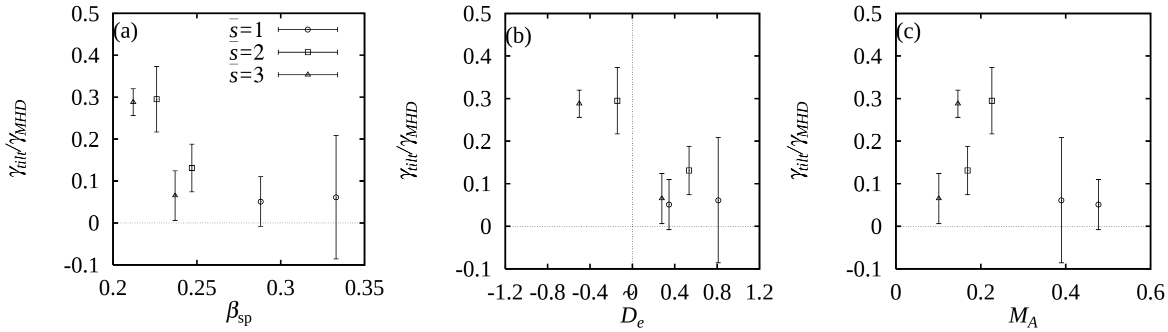

FIG. 4: *Dependence of the tilt growth rate γ_{tilt} on (a)plasma beta value $\beta_{\rm sp}$ at separatrix, (b)electron hollowness parameter $\tilde{D}_e$, and (c)Alfvén Mach number M_A. The open circle shows the case of $\overline{s} = 1$, the open square shows the case of $\overline{s} = 2$, and the open triangle shows the case of $\overline{s} = 3$, respectively.*

plasma density at the field-null line, and the magnetic field at the wall. A glance at Fig. 4 reveals that γ_{tilt} reduces to about 5% to 25% of γ_{MHD} because of the kinetic effect.

Figure 4(a) shows that γ_{tilt} tends to decrease as $\beta_{\rm sp}$ increases. This result means that the separatrix beta value is relevant to the tilt stabilization. Nishimura *et al* suggest from this tendency that the anchoring ions may play a role to connect the unstable internal plasmas with stable external plasmas and keep the system stable against the tilt instability[12].

It is worthly of notice from Fig. 4(b) that the tilt growth rate is remarkably reduced when the electron current profile is hollow ($\tilde{D}_e > 0$). Furthermore, it is important to point out that there is a clear correlation between the growth rate and the electron hollowness parameter, although all sorts of the simulation results obtained from various initial conditions are demonstrated in Fig. 4(b). These results indicate that the electron hollowness parameter has much to do with the tilt stabilization. This tendency coincides with the analysis of the experiments by Steinhauer and Ishida[4].

In the MHD simulation[2], the tilt mode is stabilized due to the spin stabilization effect when $M_A > 1$. From Fig. 4(c), on the other hand, the tilt stabilization becomes visible in the region of $M_A \approx 0.5$. Therefore this stabilization is not explained directly by the spin stabilization effect. Because the relationship between γ_{tilt} and M_A is complex, it is suggested that the ion toroidal motion partially contributes to the tilt stabilization.

These results lead us to the conclusion that the tilt mode tends to be stabilized in the cases of the hollow current profile and high separatrix beta.

5. Summary

The two-dimensional electromagnetic particle simulation is performed to investigate the profile relaxation from an MHD profile to a kinetic equilibrium and to clarify the property of the kinetic equilibrium of the field-reversed configurations independently of the tilt instability. And then we perform the three-dimensional full electromagnetic particle simulation using the kinetic profile obtained from the two-dimensional simulation as the initial condition to examine the stability of the kinetic equilibrium against the tilt mode.

The relaxation oscillation takes place when the profile relaxes from an MHD profile to a kinetic equilibrium. After this profile relaxation, the electron current profile changes to a hollow profile around the filed-null line as a result of the combined effects of the gradient-B drift near the field-null line and the $E \times B$ drift generated by the ion finite Larmor radius effect near the magnetic separatrix. On the other hand, the ion current profile becomes a peaked profile because of the effect of the ion meandering motion along the field-null line.

The growth rate of the tilt instability in all cases reduces to a small value because of the kinetic effect. In the system where the hollow current profile is realized after the profile relaxation, the growth of the tilt instability is suppressed, while in the system with peaked current profile, the tilt instability grows. From the investigation into the relationship between the tilt growth rate and several parameters, we find that the electron hollowness parameter and the separatrix beta value are important keys to solve the problem of the tilt stabilization.

Acknowledgments

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