Accessibility of Equilibrium with Shallow/Deep Penetration in Rotating Magnetic Field Current Drive

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Abstract

The accessibility to the equilibrium of the shallow/deep penetration of the RMF into an FRC is studied by numerically integrating the rate equations for the ion and electron rotations with the perturbed initial equilibrium values. When the initial angular frequency of the electron fluid is less than the equilibrium value which satisfies the stability conditions, the electron angular frequency stays near the initial value to achieve the steady equilibrium of the shallow penetration. The initial electron angular frequency barely larger than the equilibrium brings rapidly the angular frequency close to the RMF frequency and results in the full penetration of the RMF. On the other hand, when the equilibrium is unstable, the initial value smaller than the equilibrium electron rotation leads to the continuous current decay and does not sustain the configuration. The larger initial value results in the rapid full penetration of the RMF similarly to the case of the stable equilibrium. When the partially penetrated equilibrium is stable, the full penetration may not be achieved, as is observed in the present experiments, from the state far from the electron rotation synchronous with the RMF.

I. INTRODUCTION

The current drive by applying the rotating magnetic field (RMF) was first proposed by H.A. Blevin and P.C. Thoneman.¹ It was demonstrated by the device of Rotamak and produced a steady Filed Reversed Configuration.² The RMF is penetrated into a highly conducting plasma column, since the electrons in almost synchronous rotation with the RMF see an effectively very low frequency, i.e. a Doppler shifted frequency in their own frame of reference. The angular frequency of the RMF must lie in the range $\omega_{ci} < \omega << \omega_{ce}$, where $\omega_{ci} = eB_{\omega}/m_i$ is the ion cyclotron frequency with respect to the rotating field strength B_{ω} , and $\omega_{ce} = eB_{\omega}/m_e$ is the electron cyclotron frequency of the same field so that the electrons may tie the RMF and the ions may not.³ On the other hand, the flux preserving motion of the electron fluid makes the electrons being not tied to the axial field line, but tied to the rotating field line, of which strength is weaker than that of the axial field.

The previous numerical study⁴ indicated that the RMF exceeding a threshold value

penetrated completely into a plasma column. The experimental results, however, are contrary to the numerical results.^{5,6} The recent numerical simulations⁷ including the effects of the variations of the plasma density and the separatrix radius showed that the RMF penetrated barely up to the field null, which is consistent with the experiments. The penetration of the RMF was discussed at the view-point of the stability of the equilibrium with respect of the rotation rate of the rigidly rotating electron fluid. The linear stability analysis⁸ gave the stability conditions at the equilibrium of the shallow penetration of the RMF and well explained the experimental results. The dynamic behaviors of the electron angular frequency near the equilibrium are here numerically studied and the accessibility to the shallow/deep penetration are discussed.

II. BASIC EQUATIONS

We assume the rigid rotations of both of electron and ion fluids. The radial and azimuthal components of the magnetic field penetrating into the plasma are analytically given by

$$\widetilde{B}_{r} = \frac{2B_{\omega}}{\sqrt{ikr}} \frac{I_{1}(\sqrt{ikr})}{I_{0}(\sqrt{ika})} e^{i(\omega t - \theta)}$$
⁽¹⁾

$$\widetilde{B}_{\theta} = i \left(\widetilde{B}_{r} - 2B_{\omega} \frac{I_{0}(\sqrt{ikr})}{I_{0}(\sqrt{ika})} e^{i(\omega t - \theta)} \right)$$
⁽²⁾

respectively. Here, $ka = \left(\frac{a\omega_{pe}}{c}\right)\sqrt{(\omega - \omega_e)/v_{ei//}}$, *a* is the separatrix radius, ω_{pe} is the plasma frequency, *c* is the speed of light, B_{ω} is the strength of the RMF, ω is the frequency of the RMF, ω_e is the rigid rotation frequency of the electron fluids, $v_{ei//}$ is the electron-ion collision frequency parallel to the axial magnetic field.

The time variation of the ion angular frequency is given by the balance of the drag of the neutral atom and the force exerted by the RMF;

$$\frac{\partial \omega_i}{\partial t} = -\nu_{in}\omega_i + \frac{e^2}{2m_e m_i} \frac{\overline{\omega}'}{\overline{\omega}'^2 + 1} \left\langle \left| \tilde{B}_r \right|^2 \right\rangle_s$$
(3)

where $\overline{\omega}' = (\omega - \omega_e) / v_{ei//}$, v_{in} is the collision frequency between the ions and the neutral atoms. $\langle \rangle_s$ denotes the specially averaged value. When the azimuthal motion of the ion and

electron fluids is a rigid rotation, the axial magnetic field is given by

$$B_z(r) = 2B_a \left(\frac{r}{a}\right)^2 - B_a \tag{4}$$

where

$$B_a = \frac{\mu_0 en(\omega_e - \omega_i)}{4} a^2 \tag{5}$$

When we allow the radial motion of the separatrix radius and assume that the plasma is surrounded by a conducting wall of the radius r_w , the flux conservation must be satisfied, i.e.

$$\left(r_{w}^{2}-a^{2}\right)B_{a}=const.$$
(6)

Using Ohm's law and Faraday's law in addition to the flux conservation, we have the time variation of the difference between the electron and ion angular frequencies;

$$\frac{\partial(\omega_{e}-\omega_{i})}{\partial t} = -\frac{8}{\Gamma} \left(\frac{c}{a\omega_{pe}}\right)^{2} v_{ei\perp} (\omega_{e}-\omega_{i}) + \frac{4}{\Gamma} \left(\frac{c}{a\omega_{pe}}\right)^{2} \frac{\overline{\omega}'}{\overline{\omega}'^{2}+1} \frac{e^{2}}{m_{e}^{2}} \left\langle \left|\widetilde{B}_{r}\right|^{2} \right\rangle_{s}$$
(7)

where $\Gamma = \frac{r_w^2 - a^2}{2a^2 - r_w^2} > 0$. Then the separatrix radius must lie between $\frac{r_w}{\sqrt{2}} < a < r_w$.

The dynamic behaviors of the parameters such as ω_e , ω_i , ka and a are obtained by numerically integrating Eqs.(3),(5),(6) and (7) with the perturbed initial values near the equilibrium.

III. NUMRICAL RESULTS

The equilibriums are obtained by setting the Eqs.(3) and (7) zero. The equilibrium rotations of the ions is given by

$$\omega_{i0} = \frac{1}{\alpha + 1} \omega_{e0} \tag{8}$$

where $\alpha = (v_{in}/v_{ei})(m_i/m_e) >> 1$. Table 1 shows the parameters used for the numerical studies. Figure 1 shows the equilibrium solutions of the electron fluid. The solutions are given by the intersecting points of two curves representing the ion drag and the force driven by the RMF. The solutions shown by the white and black circles correspond the partial and full penetrations of the RMF, respectively.

We examine the dynamic behaviors of the parameters near the equilibrium of the shallow penetration. Figure 2 shows the time variations of the electron angular frequency, the ion angular frequency, the separatrix radius and the penetration parameter of *ka* initiated by giving the initial values near the equilibrium of the partial penetration. The parameters used in the calculations are the same as Table 1 except a = 0.2m and satisfy the stability condition given by Eq.(37) in the paper⁸.

Figure 3 shows the same dynamic behaviors as Fig.2 for the larger separatrix radius, i.e. a = 0.3m which makes the shallow equilibrium unstable. The initial electron angular frequency higher than the equilibrium leads to the equilibrium of the full penetration of the RMF. The lower electron angular frequency results in the continuous decreases of the rotation angular frequency of both of the ions and electrons, i.e. no equilibrium is sustained.

IV. CONCLUSIONS

The accessibility to the equilibrium with the shallow/deep penetration of the RMF into an FRC is studied by numerically integrating the rate equations for the ion and electron rotations with the perturbed initial equilibrium values. When the initial angular frequency of the electron fluid is less than the equilibrium value which satisfies the stability conditions for the rotation rate of the electrons and ions, the electron angular frequency stays near the initial value to achieve the equilibrium of the shallow penetration. The initial electron angular frequency barely larger than the equilibrium brings rapidly to the angular frequency close to the RMF frequency and results in the full penetration of the RMF.

On the other hand, when the equilibrium is unstable, the smaller initial value than the equilibrium electron rotation leads to the continuous current decay and does not sustain the The larger initial value results in the rapid full penetration of the RMF configuration. similarly to the case of the unstable equilibrium. Both of the perturbed initial values do not keep the electron angular frequency near the initial equilibrium value. When the equilibrium, therefore, is unstable, only the equilibrium of the full penetration may be achieved. However, when the equilibrium is stable, the full penetration cannot be accessed, as is observed in the present experiments, from the electron rotating state non-synchronous with the RMF. The study on the dynamic behaviors of the electron angular frequency clarifies the accessibility to the equilibrium of the shallow/deep penetration and the means to control the penetration of the RMF.

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TABLE I. Reference Parameters.

$B_{\omega} = 40 G$	$v_{ei//} = 1.1 \times 10^7 s^{-1}$
$\omega = 0.5 \times 10^6 r/s$	$\overline{v}_{in} = 0.012$
$T_e = 15 eV$	$a\omega_{_{pe}}/c = 150.0$
$n = 1.0 \times 10^{19}$ m^{-3}	$\overline{\omega}_{rf} = 64.5$
a = 0.25 m	$\overline{\omega} = 0.045$
$r_w = 0.31 m$	$\alpha = 21.7$
$B_{z} = 160 G$	$\Gamma = 1.0$
	$v_{ei\perp}/v_{ei//}=1$



Fig.1 Equilibrium solutions of electron fluid with a rigid rotation for (a) a = 0.2m and (b) 0.3m. The solutions shown by the white and black circles indicate the partial and full penetrations of the RMF, respectively.



Fig.2 (a)





Fig.2 (c)

Fig.2 (d)

Fig.2 Dynamic behaviors of (a) the electron angular frequency, (b) the ion angular frequency, (c) the electron angular frequency and (d) the synchronous parameters of ka initiated by perturbed initial values around the stable equilibrium (a = 0.2m), respectively.



Fig.3 (a)

Fig.3 (b)



Fig.3 (c)

Fig.3 (d)

Fig.3 Dynamic behaviors of (a) the electron angular frequency, (b) the ion angular frequency, (c) the electron angular frequency and (d) the synchronous parameters of ka initiated by perturbed initial values around the unable equilibrium (a = 0.3m), respectively.