Proof of non-invariance of magnetic helicity in ideal plasmas and a general theory of self-organization for open and dissipative dynamical systems

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It is proved that the global magnetic helicity is not invariant, even in an ideally conducting MHD plasma. A novel general theory is presented in which a variety of self-organized states in open and dissipative dynamical systems with various fluctuations can be found. This theory is based on the principle that the self-organized states must be those states for which the rate of change of global auto-correlations for multiple dynamical field quantities, which depend on multidimensional mutually independent variables, is minimized. One of the important points of this theory is that the original generalized dynamic equations are embedded in the final equivalent definition for the self-organized states, and therefore the equations deduced from the final equivalent definition include all the time evolution characteristics of the dynamical system of interest. Since states derived from the Euler-Lagrange equations with the use of variational calculus have minimal rates of change of the global auto-correlations, they are most stable and unchangeable compared with other states.
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After J. B. Taylor published his famous theory[1] to explain the appearance of the reversed field pinch configuration[2], global magnetic helicity has been believed to play an important role as a global invariant in the self-organization process and relaxation phenomena of magnetized plasmas. However, another model (a partially relaxed state model) has been proposed to explain the reversed field pinch experimental data[3–7]; and the partially relaxed state model and the mode transition point of the self-organized state were deduced from the energy integral, without the assumption that the global magnetic helicity is invariant. A subsequent version of this theory of self-organization was developed, based on auto-correlations of physical quantities, which includes the Taylor state as a limiting case[8,9]. In the present paper, we prove that the global magnetic helicity is not an invariant even in ideally conducting MHD plasmas and that therefore the Taylor relaxation process never occurs physically in real experimental plasmas and simulations. Furthermore, we present a novel general theory for how to find self-organized states in open and dissipative dynamical systems. This theory is applicable to various nonlinear dynamical systems and reproduces the Taylor state as a limiting case[5–12]. We also show some applications of the present theory to dissipative Korteweg-deVries solitons, incompressible viscous fluids and dissipative MHD plasmas.

J. B. Taylor's theory is based on Maxwell's equations for the electric and magnetic fields. He has introduced the global magnetic helicity K[1], defined in a volume V bounded by an ideally conducting surface, and derived the time derivative of K, as follows:

$$K \equiv \int_V \mathbf{A} \cdot \mathbf{B} \, \mathrm{d}V \, . \tag{1}$$

$$\frac{\partial K}{\partial t} = -\frac{2}{\mu_0} \int_V \mathbf{E} \cdot \mathbf{B} \mathrm{d}V + \frac{1}{\mu_0} \oint_S (\mathbf{E} \times \mathbf{A}) \cdot \mathrm{d}\mathbf{S}, \tag{2}$$

Taylor's theory involves the following two conjectures of (A) and (B): (A) Since magnetic fields are frozen in an ideal MHD plasma during its local flow, the global magnetic helicity K is considered to be conserved as a topolo-gical quantity of the magnetic field lines. By using the simplified Ohm's law of $\eta \mathbf{j} = \mathbf{E} + \mathbf{v} \times \mathbf{B}$ and putting resistivity $\eta = 0$, one finds that the volume integral of $\mathbf{E} \cdot \mathbf{B}$ for every flux tube is zero. Then the volume integral term in Eq.(2) vanishes. Since $\mathbf{E} \cdot \mathrm{d}\mathbf{S}$ is zero at an ideally conducting surface, the time derivative of K is zero in an ideal MHD plasma. Thus, by this argument, K is considered to be conserved as an invariant in ideal MHD plasmas. (B) When the resistivity η is small but finite, reconnection of magnetic field lines can take place. However, it is conjectured that the global helicity K can be treated as an invariant during the relaxation process in a non-ideal MHD plasma, because the resistive decay of the total magnetic energy inside an ideally conducting wall is faster than the decay of K. Using variational calculus with the global constraint K = const., J. B. Taylor derived the relaxed state of $\nabla \times \mathbf{B} = \lambda \mathbf{B}$ from the Euler-Lagrange equation[1]. This result had been previously obtained by S. Chandrasekhar and L. Woltjer for states with minimum dissipation of magnetic energies, also with the use of variational calculus[13]. Taylor's logic, described in the preceding paragraph, is, however, not based on either a variational principle or an energy principle.

It is commonly known that the use of either a variational principle[14] or an energy principle[15] leads to dynamical equations that give the time evolution of the dynamical system of interest, as shown in classical mechanics theory[14] and the well-known dynamical equations for perturbed elements in an ideal MHD plasma[15].

We now prove that both of two Taylor's conjectures (A) and (B) shown above, are not physically accessible. In order to find the time change of K in an ideal MHD plasma, we must return to Eq.(2) and check the volume integral term more carefully. From the scalar product of the generalized Ohm's law for a fully ionized plasmas in the limit of zero resistivity η, we can derive the volume integral term of Eq.(2) as follows,

$$\frac{2}{\mu_0}\int_V \mathbf{E}\cdot\mathbf{B}\mathrm{d}\mathbf{V} = \frac{\mathbf{2}}{\mu_\mathbf{0}}\int_\mathbf{V}\{[\frac{\mathbf{c^2}}{\omega_\mathrm{pe}^\mathbf{2}}\frac{\partial\mathbf{j}}{\partial\mathbf{t}} - \frac{\mathbf{1}}{\mu_\mathbf{0}\mathbf{e}\mathbf{n}_\mathrm{e}^\mathbf{2}}(\mathbf{p}_\mathrm{e} - \frac{\mathbf{m}_\mathrm{e}\mathbf{Z}}{\mathbf{m}_\mathrm{i}}\mathbf{p}_\mathrm{i})\nabla\mathbf{n}_\mathrm{e}]\cdot\mathbf{B}\}\mathrm{d}\mathbf{V}, \tag{3}$$

where the usual notations for plasma physics quantities, such as the number density of electrons n_e, and electron and ion pressures p_e and p_i, are used. Also, the boundary conditions $\mathbf{B}\cdot\mathrm{d}\mathbf{S} = 0$ and $\nabla\cdot\mathbf{B} = 0$ at an ideally conducting wall were used in obtaining Eq. (3). Since all the volume integral terms on the right-hand side of Eq. (3), determined by local physical quantities, can usually have either positive or negative values for turbulent plasmas, we easily confirm that the time rate of change of K can be either positive or negative (the possibility of being zero is statistically negligible). Therefore one cannot definitely conclude that K is a physical invariant, even within an ideal MHD plasma. Since K is never invariant during local plasma flows or relaxation even in an ideal MHD plasma, the global constraint that uses K has no power to limit the relaxation process itself within an MHD plasma. The mathematical procedure in conjecture (B) is simple calculus to find a group of solutions having minimum magnetic energy within a wider set of solutions having the same value of K. We should notice that after the configuration of the magnetic field lines in a plasma is determined, the value of a topological quantity can be calculated, but the value itself has no power inversely to determine the configuration of field lines. Without using topological quantities such as K, we are able to derive the Taylor state of $\nabla\times\mathbf{B} = \lambda\mathbf{B}$, starting from the fundamental definition of the self-organized states[9].

Owing to Taylor's theory, several theories have appeared to use topological invariants and have minimized energy or maximize entropy to obtain self-organized states. However, as is proved above, those topological invariants have no power to determine the relaxed states, and therefore we need other logical principle for new self-organization theory applicable to any dissipative dynamical systems.

We develop here a novel basic formulation of a general theory to find self-organized states that is an extension of the theory in Ref.[9]. It should be emphasized that the present theory, which uses auto-correlations for dynamical quantities, is not based on either a variational principle or an energy principle, and also that the global auto-correlations are not time invariants.

We consider a set of N dynamical variables $\mathbf{q}\equiv\mathbf{q}[\xi^k]\equiv(q_1[\xi^k],\ldots,q_N[\xi^k])$, with M-dimensional independent variables $[\xi^k]$ $(k = 1,2,\ldots,M)$, which may include time, space, and velocity in distribution functions, or prices, amount of materials, budgets for production systems, and other such various variables. Using generalized symbolic dynamical operators, we may write the general nonlinear N-set simultaneous equations for an open or a closed dynamical system as

$$\partial q_i[\xi^k]/\partial\xi^j \;=\; D_i^j[\mathbf{q}]\;, \tag{4}$$

where $D_i^j[\mathbf{q}]$ $(j = 1,2,\ldots,N)$ represents dissipative or non-dissipative, linear or nonlinear operators for the change of a dynamical variable q_i along an independent variable ξ^j. After multiplying $q_i[\xi^k]$ on both sides of Eq. (4) and integrating over the independent variables $[\xi^k]$ $(k\neq j)$, we obtain "conservation laws" for all the dynamical variables q_i as follows:

$$\frac{\partial}{\partial\xi^j}\int\frac{1}{2}q_i^2|J_{k\neq j}|\prod_{k\neq j}\mathrm{d}\xi^k = \int q_iD_i^j[\mathbf{q}]|J_{k\neq j}|\prod_{k\neq j}\mathrm{d}\xi^k. \tag{5}$$

Here, the dynamical system of interest always has fluctuations of the dynamical variables $q_i[\xi^k]$ along the axis of the variable ξ^j. The fluctuations may have several characteristic lengths in different orders along ξ^j, one of which is expressed as τ_{ci}. The characteristic length τ_{ci} may give the ordering of the relaxation time scale. From the standpoint of observations, the self-organized relaxed states are identified by the following definition (6), hereafter noted as Def. (6), with the use of auto-correlations between the dynamical variables $q_i[\xi^j]$ and $q_i[\xi^j+(\triangle\xi^j/\tau_{ci})]$, where the increase of ξ^j for q_i is normalized by τ_{ci}, as

$$\min\;\left|\;\frac{\int q_i[\xi^j]q_i[\xi^j+(\triangle\xi^j/\tau_{ci})]|J_{k\neq j}|\prod_{k\neq j}\mathrm{d}\xi^k}{\int(q_i[\xi^k])^2|J_{k\neq j}|\prod_{k\neq j}\mathrm{d}\xi^k} - 1\,\right|. \tag{6}$$

Using Taylor expansion for Def. (6), we obtain the following equivalent definition Def. (7) for the self-organized states from the first order of $\triangle\xi^j/\tau_{ci}$. Substituting the original dynamical equations (4) into Def. (7), we obtain the final definition Def. (8) for the self-organized states.

$$\min \ \left| \frac{\int q_i[\xi^j](\partial q_i[\xi^j]/\partial\xi^j)|J_{k\neq j}|\prod_{k\neq j}\mathrm{d}\xi^k}{(\tau_{ci}/\triangle\xi^j)\int(q_i[\xi^k])^2|J_{k\neq j}|\prod_{k\neq j}\mathrm{d}\xi^k} \right| . \tag{7}$$

$$\min \ \left| \frac{\int q_i[\xi^j]D_i^j[\mathbf{q}]|J_{k\neq j}|\prod_{k\neq j}\mathrm{d}\xi^k}{(\tau_{ci}/\triangle\xi^j)\int(q_i[\xi^k])^2|J_{k\neq j}|\prod_{k\neq j}\mathrm{d}\xi^k} \right| . \tag{8}$$

It should be emphasized that all of the dynamical laws, characterized by the nonlinear simultaneous system of equations, Eq. (4), are embedded in the equivalent definition, Def. (8). Since the nonlinear set of N simultaneous equations, Eq. (4), connect mutually the set of N dyamical variables $q_i[\xi^k]$ $(i = 1, 2, \ldots, N)$, the mathematical expressions of Defs. (7) and (8) are obtained by variational calculus with the use of functionals F with Lagrange multipliers λ_i as follows

$$F = \int \sum_i \{ \frac{1}{2}\frac{\partial q_i[\xi^k]^2}{\partial\xi^j} + \frac{\tau_{ci}}{\triangle\xi^j}\lambda_i q_i[\xi^k]^2 \} |J_{k\neq j}| \prod_{k\neq j}\mathrm{d}\xi^k, \tag{9}$$

$$F = \int \sum_i \{ q_i[\xi^k]D_i^j[\mathbf{q}] + \frac{\tau_{ci}}{\triangle\xi^j}\lambda_i q_i[\xi^k]^2 \} |J_{k\neq j}| \prod_{k\neq j}\mathrm{d}\xi^k, \tag{10}$$

$$\delta F = \int \sum_i \{ \delta q_i[\xi^k](D_i^j[\mathbf{q}] + \frac{\tau_{ci}}{\triangle\xi^j}\lambda_i q_i[\xi^k]) + q_i[\xi^k]\delta(D_i^j[\mathbf{q}] + \frac{\tau_{ci}}{\triangle\xi^j}\lambda_i q_i[\xi^k]) \} |J_{k\neq j}| \prod_{k\neq j}\mathrm{d}\xi^k = 0, \tag{11}$$

$$\delta^2 F = \int \sum_i \{ \delta q_i[\xi^k](\delta D_i^j[\mathbf{q}] + \frac{\tau_{ci}}{\triangle\xi^j}\lambda_i \delta q_i[\xi^k]) \} |J_{k\neq j}| \prod_{k\neq j}\mathrm{d}\xi^k \ \geq 0, \tag{12}$$

where the functional given in Eq. (10) is used in the first step; δF and $\delta^2 F$ are, respectively, the first and the second variations of F; and the variational calculus is performed with respect to the dynamical variables q_i that depend on the variables ξ^k except $k = j$. By means of repeated partial integration and application of the boundary conditions, we eventually obtain the simplest expression for the terms of the operator $D_i^j[\mathbf{q}]$, which we denote by $D_i^{j\#}[\mathbf{q}]$; this reduction had been previously reported in[7,8]. In terms of this notation, the condition for the marginal minimum for arbitrary variation $\delta q_i[\xi^k_{k\neq j}]$ is given by Eq. (13) from Eq. (12) as follows,

$$\delta D_i^{j\#}[\mathbf{q}] + (\tau_{ci}/\triangle\xi^j)\lambda_i \delta q_i[\xi^k] \ = 0 \, . \tag{13}$$

Substituting Eq. (13) into Eq. (11), we obtain the Euler-Lagrange equation, Eq. (14), for arbitrary variation δq.

$$D_i^{j\#}[\mathbf{q}] + (\tau_{ci}/\triangle\xi^j)\lambda_i q_i[\xi^k] \ = 0 \, . \tag{14}$$

Equation (14) can be written as the eigenvalue equation with boundary conditions for $\delta q_i[\xi^k_{k\neq j}]$, viz., $D_i^{j\#}[\mathbf{u}]$ $+(\tau_{ci}/\triangle\xi^j)\lambda_{im}u_{im}[\xi^k] = 0$, where $u_{im}[\xi^k]$ and λ_{im} are the normalized eigenvalue solutions and their eigenvalues, respectively, with the appropriate normalization written as $\int u_{im}[\xi^k] \ u_{in}[\xi^k]|J_{k\neq j}|\prod_{k\neq j}\mathrm{d}\xi^k = \delta_{mn}$, as was reported in[7,8]. Substituting one of these eigensolutions into Eq. (12) and using the eigenvalue equation, we obtain the following

$$\delta^2 F = \sum_i \frac{\tau_{ci}}{\triangle\xi^j}(\lambda_{im} - \lambda_i) \int (u_{im}[\xi^k])^2 |J_{k\neq j}| \prod_{k\neq j}\mathrm{d}\xi^k \ \geq 0 \, , \tag{15}$$

Since Eq. (15) is required for all eigenvalues, we obtain the following condition for the self-organized state with the minimum rate of change:

$$0 \ < \lambda_i \ < \ \lambda_{i1} \, , \tag{16}$$

where λ_{i1} is the smallest positive eigenvalue and λ_i is taken to be positive. On the other hand, when we use Eq. (9) in the second step, we obtain another functional F and its first variation δF, Eq. (17), beeing equivalent to Eqs. (10) and (11), their Euler-Lagrange equations, Eq. (18), for arbitrary variations $\delta q_i[\xi^k_{k\neq j}]$, and their final solutions, Eq. (19), as follows:

$$\delta F = \int \sum_i \{\delta q_i[\xi^k_{k\neq j}](\partial q_i[\xi^k_{k\neq j}]/\partial\xi^j + \frac{\tau_{ci}}{\triangle\xi^j} 2\lambda_i q_i[\xi^k_{k\neq j}])\}|J_{k\neq j}| \prod_{k\neq j} \mathrm{d}\xi^k = 0, \tag{17}$$

$$\partial q_i[\xi^k_{k\neq j}]/\partial\xi^j + \frac{\tau_{ci}}{\triangle\xi^j} 2\lambda_i q_i[\xi^k_{k\neq j}] = 0\ , \tag{18}$$

$$q_i[\xi^k_{k\neq j}] = \exp(-2\lambda_i\xi^j)\ q_i[\xi^j_0, \xi^k_{k\neq j}]\ , \tag{19}$$

where ξ^j_0 is the initial value of ξ^j at the self-organized states.

The new theory presented above is a natural logical extension of the theories reported in[7,8,10] to the general nonlinear set of N simultaneous equations expressed by Eq. (4),

When we apply the general theory to the Korteweg-deVries equation of Eq. (20) with a dissipative viscosity ν term, we obtain the following analytical self-organized soliton solutions of Eq. (21) from Eq. (19)

$$\partial u/\partial t = -6u(\partial u/\partial x) - \partial^3 u/\partial x^3 + \nu\partial^2 u/\partial x^2\ . \tag{20}$$

$$u^*(t,x) = \exp[-2\lambda t \pm i(\lambda/\nu)^{1/2}]\ u^*(t_0, x_0)\ . \tag{21}$$

It should be empasized here that the analytical solution Eq. (21) coincides very well with the simulations reported in[11].

Second application is to another dynamical system of the incompressible viscous fluid written by Eq. (22) with the periodic boundary condition with edge length 1 in x, y plane, and have obtained analytical solution for the self-organized state $\boldsymbol{\omega}^*$, Eq. (23), as

$$\partial\boldsymbol{\omega}/\partial t = -(\mathbf{u}\cdot\nabla)\boldsymbol{\omega} + \nu\nabla^2\boldsymbol{\omega}, \tag{22}$$

$$\boldsymbol{\omega}^*(t,x,y) = \exp(-4\pi^2\nu t)[\cos 2\pi x_0 + \cos 2\pi y_0]\mathbf{k}, \tag{23}$$

where $\boldsymbol{\omega} = \nabla\times\mathbf{u}$, $\nabla^2\psi = \omega$, and the relation of ω and the flow function ψ at the relaxed state is given by $\omega = 4\pi\psi$. Using computer simulations with quite long effective computation time compared with the simulation data in[18] more than 10 times longer, we have confirmed that the analytical solution of Eq. (23) agrees very well with numerical data, which will be appear elsewhere.

Third application is to compressible resistive and viscid MHD fusion plasmas. Using the generalized Navier-Stokes equation, Faraday's law, the energy conservation, and the generaliec Ohm's law , we obtain the following nonlinear simultaneous dynamical equations, i. e., the three power balance equations for kinetic flow energy, magnetic field enegy, and internal thermal energy:

$$\begin{aligned}\frac{\partial}{\partial t}\frac{\rho_{\rm m}\mathbf{u}\cdot\mathbf{u}}{2} = \mathbf{u}\cdot[-\frac{\mathbf{u}}{2}\nabla\cdot(\rho_{\rm m}\mathbf{u}) + \rho\eta\mathbf{j} + (\frac{\rho}{en}+1)(\mathbf{j}\times\mathbf{B}-\nabla p) + \frac{m_{\rm e}\rho}{{\rm e}^2 n}\frac{\partial\mathbf{j}}{\partial t} - \nu\nabla\times\nabla\times\mathbf{u} \\ +(4/3)\nu\nabla(\nabla\cdot\mathbf{u}) + (1/3)\nabla\nu(\nabla\cdot\mathbf{u}) + \nabla(\nabla\nu\cdot\mathbf{u}) - \nabla\times(\nabla\nu\times\mathbf{u}) - \mathbf{u}\nabla^2\nu].\end{aligned} \tag{24}$$

$$\frac{\partial}{\partial t}\frac{\mathbf{B}\cdot\mathbf{B}}{2\mu_0} = \frac{\mathbf{B}}{\mu_0}\cdot\{\nabla\times[\mathbf{u}\times\mathbf{B} - \eta\mathbf{j} - \frac{1}{en}(\mathbf{j}\times\mathbf{B}-\nabla p) - \frac{m_{\rm e}\rho}{{\rm e}^2 n}\frac{\partial\mathbf{j}}{\partial t}]\}. \tag{25}$$

$$\frac{\partial}{\partial t}(\frac{p}{\gamma-1}) = -\frac{p}{\gamma-1}[\nabla\cdot(p\mathbf{u}) + (\gamma-1)p\nabla\cdot\mathbf{u}] + \mathbf{j}\cdot[\eta\mathbf{j} - \frac{1}{en}\nabla p + \frac{m_{\rm e}}{{\rm e}^2 n}\frac{\partial\mathbf{j}}{\partial t}] + \nabla\cdot(\kappa\nabla T). \tag{26}$$

Using the condition for the marginal minimun Eq. (13), and resumitting this condition into the 1st variation δF, we obtain the three Euler-Lagrange equations for the three dynamical equations:

$$\begin{aligned}\frac{\mathbf{u}}{2}\nabla\cdot(\rho_{\rm m}\mathbf{u}) - \rho\eta\frac{1}{\mu_0}\nabla\times\mathbf{B} + (\frac{\rho}{en}+1)(\frac{\mathbf{B}}{\mu_0}\times\nabla\times\mathbf{B} + \nabla p) - \frac{m_{\rm e}\rho}{\mu_0{\rm e}^2 n}\nabla\times\frac{\partial\mathbf{B}}{\partial t} + \nu\nabla\times\nabla\times\mathbf{u} \\ +\nabla\times(\nabla\nu\times\mathbf{u}) - \frac{4}{3}\nu\nabla(\nabla\cdot\mathbf{u}) - \frac{1}{3}\nabla\nu(\nabla\cdot\mathbf{u}) - \nabla(\nabla\nu\cdot\mathbf{u}) + \mathbf{u}\nabla^2\nu = \lambda_{\rm u}\frac{\rho_{\rm m}\mathbf{u}}{2}.\end{aligned} \tag{27}$$

$$\nabla\times[\frac{\eta}{\mu_0}\nabla\times\mathbf{B}-\mathbf{u}\times\mathbf{B}-\frac{1}{en}(\frac{\mathbf{B}}{\mu_0}\times\nabla\times\mathbf{B}+\nabla p)+\frac{m_{\rm e}\rho}{\mu_0{\rm e}^2n}\nabla\times\frac{\partial\mathbf{B}}{\partial t}]=\lambda_{\rm B}\mathbf{B}. \tag{28}$$

$$\frac{p}{\gamma-1}[\nabla\cdot(p\mathbf{u})+(\gamma-1)p\nabla\cdot\mathbf{u}]-\frac{\eta}{\mu_0}(\nabla\times\mathbf{B})\cdot(\frac{\eta}{\mu_0}\nabla\times\mathbf{B}-\frac{1}{en}\nabla p+\frac{m_{\rm e}}{\mu_0{\rm e}^2n}\frac{\partial\mathbf{j}}{\partial t})-\nabla\cdot(\kappa\nabla T)=\lambda_{\rm p}(\frac{p}{\gamma-1}). \tag{29}$$

An important point of the self-organized states, derived by the present theory shown above, is that the configurations of the relaxed states gradually change in time, and the changed states shall become unstable to repeat again the relaxation process, just as observed in experimental fusion plasmas and the simulation data which have been reported so far. From Eq. (28), we obtain $\nabla\times\nabla\times\mathbf{B}=(\mu_0/\eta)\lambda_{\rm B}\mathbf{B}$ for a limiting case with uniform resistivity, $\mathbf{u}=0$ and quasi-steady pressureless plasma. This result includes the Taylor state of $\nabla\times\mathbf{B}=\lambda\mathbf{B}$.

In conclusions, we have proved that the global magnetic helicity is not invariant, even in an ideal MHD plasma, and that therefore the global constraint using helicity has no power to limit the relaxation process to lead to any self-organized states in plasmas. We have also established a new general theory to find self-organized states, which includes all dynamical laws given by the nonlinear simultaneous general dynamic equations, Eq. (4), because those equations are embedded in the formulation. Since the general theory leads relaxed states haveing minimal rates of change of the global auto-correlations, they are most stable and unchangeable compared with other states. We have shown three applications to demonstrate that the new general theory of self-organization is very useful for various dynamical systems.

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