Global Eigenmodes of Low Frequency Waves in FRCs

Naotaka Iwasawa[1)], Shigefumi Okada[2)], and Seiichi Goto[2)]

1) Satellite Venture Business Laboratory, Gunma University
e-mail: iwasawa@kdlab.el.gunma-u.ac.jp
2) Plasma Physics Laboratory, Osaka University

Abstract

Global eigenmodes of low frequency waves in FRC plasmas have been obtained using MHD model and one-dimensional equilibrium model. Dispersion relation and radial structure of the global wave fields are shown for the azimuthal mode number $m = 0$. The results are compared with the results of a low frequency wave heating experiment. A possibility of ion heating by the *transit-time magnetic damping* is discussed.

1. Introduction

Low frequency waves have been used for plasma heating. Recently a heating experiment has been performed in Osaka University [1]. In this experiment, low frequency (compared with the ion cyclotron frequency in the external magnetic field) oscillating magnetic field was applied to the FRC plasma. The applied field was homogeneous in the azimuthal direction. As a result, a fluctuation of the magnetic field was observed to propagate in the direction parallel to the equilibrium magnetic field of the FRC plasma. In addition, an increase in the plasma energy was observed. Comparison between the total temperature and the ion temperature suggests that the increase in the plasma energy was mostly due to the increase in the ion temperature. This implies that the applied oscillating magnetic field could excite low frequency waves and the wave energy was absorbed by the ions. Surprisingly, toroidal field (not fluctuating) was generated by the applied magnetic field [2]. However the mechanism for these phenomena have been unclear. In this study, global eigenmodes of low frequency waves in one-dimensional FRC plasmas are analyzed for the azimuthal mode number $m = 0$ using MHD model. The results are compared with the experiments. Finally, a possibility of ion heating and the toroidal field generation by the *transit-time magnetic damping* is discussed.

2. Global eigenmode analysis

To investigate the low frequency waves propagating through FRC plasmas, the set of single-fluid MHD equations (with Hall term) is used:

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\mathbf{v})=0$$
$$\rho\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)\mathbf{v}=\mathbf{j}\times\mathbf{B}-\nabla p$$
$$\mathbf{E}+\mathbf{v}\times\mathbf{B}=\frac{1}{en}\left(\mathbf{j}\times\mathbf{B}\right)$$
$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}$$
$$\nabla\times\mathbf{B}=\mu_0\mathbf{j}$$
$$\nabla\cdot\mathbf{B}=0$$
$$\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)p\rho^{-\gamma}=0$$

Linearizing these equations and assuming that the FRC plasma has no equilibrium flow, we have 6 independent equations for 6 perturbed quantities $\mathbf{v}_1$ and $\mathbf{E}_1$. Since we seek waves propagating in the θ and z-directions *with a global radial extension* between the geometric axis and the conducting wall, the perturbed quantities are assumed to have the following form in the cylindrical coordinate system:

$$f_1(r,\theta,z,t)=\tilde{f}_1(r)\exp[i(m\theta+kz-\omega t)]$$

where m, k, and ω are the azimuthal mode number, wave number in the z-direction, and the frequency. This leads to an eigenvalue problem where the wave number k is the eigenvalue and the radial profile of the wave fields $i\tilde{v}_{1r}(r),\tilde{v}_{1\theta}(r),\tilde{v}_{1z}(r),\tilde{E}_{1r}(r),i\tilde{E}_{1\theta}(r),i\tilde{E}_{1z}(r)$ are the eigenfunctions for a given real frequency ω and a boundary conditions at $r=0$ (geometric axis) and $r=r_w$ (perfectly conducting wall). Since only propagating waves are considered in this study, k and ω are real. The problem is solved in the following way. Each eigenfunction is approximated in terms of a finite series of basis functions $\phi_n(r)$ and a function which satisfies the boundary conditions. For example, $i\tilde{v}_{1r}$ is expressed as

$$i\tilde{v}_{1r}(r)=F_{vr}(r)\sum_{n=1}^{N}C_n^{(vr)}\phi_n(r)$$
$$\phi_n(r)=\cos\left[(n-1)\pi\, r/r_w\right]$$

where $F_{vr}(r)$ is the function satisfying the boundary condition for $i\tilde{v}_{1r}(r)$, and $C_n^{(vr)}$ are the expansion coefficients. In deriving the equations for the expansion coefficients, we used the *Galerkin method*. Only modes with the azimuthal mode number $m=0$ are considered in this study.

3. One-dimensional FRC equilibria

In the perturbed equation of motion, $d^2 p_0(r)/dr^2$ appears, where $p_0(r)$ is the equilibrium plasma pressure. For the pressure profile as in Ref. [3], $d^2 p_0(r)/dr^2$ is discontinuous at the separatrix. Since continuity of $d^2 p_0(r)/dr^2$ at the separatrix is needed in this linear analysis, one-dimensional FRC equilibria have been obtained by solving the Grad-Shafranov equation for the following pressure profile

$$p(\psi) = \begin{cases} \alpha_1 p_s\left(\psi + \alpha_1\psi^2 + \alpha_2\psi^3\right) + p_s, & \psi \ge 0 \\ \alpha_1 p_s \exp(\alpha_1\psi), & \psi < 0 \end{cases}$$

where p_s is the pressure at the separatrix, α_1, α_2 are parameters. Figure 1 shows 3 different equilibria with peaked ($h = 1.2$), flat ($h = 1.0$), and hollow ($h = 0.8$) current profile. Here h is the current profile index [4]. In all these equilibria, the magnetic null and the separatrix are on $r = r_{null} = 0.32 r_w$ and $r = r_s = 0.45 r_w$.

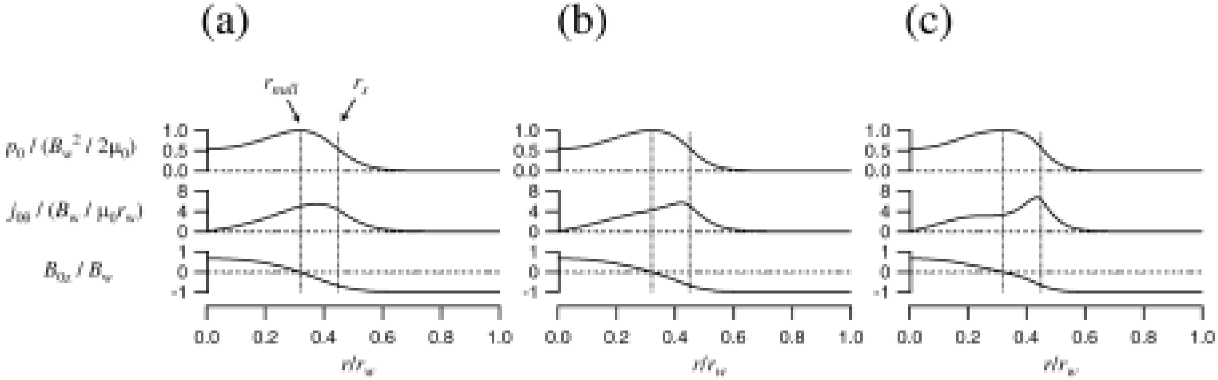

Fig. 1. Radial profile of the pressure, current density, and the magnetic field in the one dimensional FRC equilibria with (a) peaked current profile ($h = 1.2$); (b) flat current profile ($h = 1.0$); (c) hollow current profile ($h = 0.8$). $r_{null} (= 0.32 r_w)$ and $r_s (= 0.45 r_w)$ show the radii of the magnetic null and the separatrix.

4. Results

The eigenvalue problem was solved for the azimuthal mode number $m = 0$ in the FRC equilibria shown in Fig. 1. Figure 2a shows the dispersion relation of the low frequency waves propagating in *z*-direction in the peaked current equilibrium. The frequency is normalized to $\omega_{ci0} \equiv eB_w / m_i$, which is the ion cyclotron frequency in the external magnetic field. The normalized wave numbers (eigenvalues) $r_w k$ are plotted for ω/ω_{ci0} = 0.015 – 0.3 in increments of 0.015. The broken line shows the Alfvén velocity $v_{A0} \equiv B_w / (\mu_0 \rho_{null})^{1/2}$, where ρ_{null} is the mass density at the magnetic null. The solid line shows $\omega/\omega_{ci0} = 0.26$, which corresponds to 80 [kHz] (the frequency of the applied field in the experiment) for B_w =0.04[T] and deuterium plasma. Figures 2b and 2c show the dispersion relations in the flat and hollow current equilibria. We see from Fig. 2a-c that in all these equilibria the eigenmodes are dense for $\omega/k > v_{A0}$ and sparse for $\omega/k < v_{A0}$.

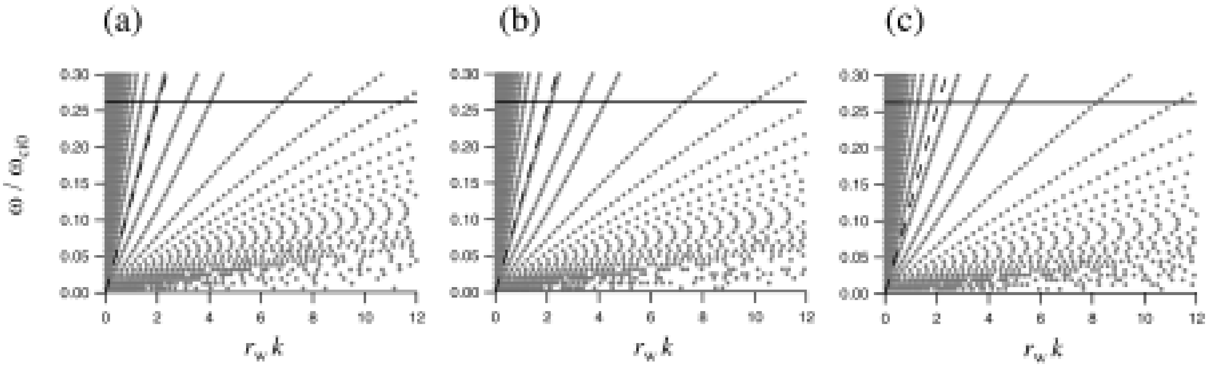

Fig. 2. Dispersion relation of the low frequency waves in the equilibria with (a) peaked current profile; (b) flat current profile; (c) hollow current profile. The broken line shows the Alfvén velocity $v_{A0} \equiv B_w / (\mu_0 \rho_{null})^{1/2}$ *, where* ρ_{null} *and* B_w *are the mass density at magnetic null (* $r = r_{null}$ *) and the magnetic field at the wall (* $r = r_w$ *).* $\omega_{ci0} \equiv eB_w / m_i$ *is the ion cyclotron frequency at the wall. The solid line shows* $\omega/\omega_{ci0} = 0.26$ *, which corresponds to 80 [kHz] (the frequency of the applied field in the experiment) for* B_w *=0.04[T] and deuterium plasma.*

Figure 3 shows the radial structure of the perturbed electric field, mass flux density, and magnetic field (the eigenfunctions) of the eigenmodes with three different wave numbers (eigenvalues) $r_w k$ on $\omega/\omega_{ci0} = 0.26$. For the phase velocity ω/k much larger than v_{A0} (see Fig. 3a), the amplitudes of the wave fields appear mainly outside the separatrix. The electric field has only *r*-component. For the mass flux density and magnetic field, θ-component is largest in $r > r_s$ and comparable to *z*-component in $r < r_s$. As the phase velocity decreases, the amplitudes move to the inner region (see Fig. 3b), and finally a mode, which has significant B_z amplitude in $r_{null} \le r \le r_s$, appears for $\omega/k = v_{A0}$ (see Fig. 3c). For phase velocity $\omega/k < v_{A0}$ mode shown in Fig. 4a appears. In that mode the amplitude of mass flux density appear between inside and outside the magnetic null. For more smaller phase velocities, modes shown in Figs. 4b,c appear. In those modes, as the phase velocity

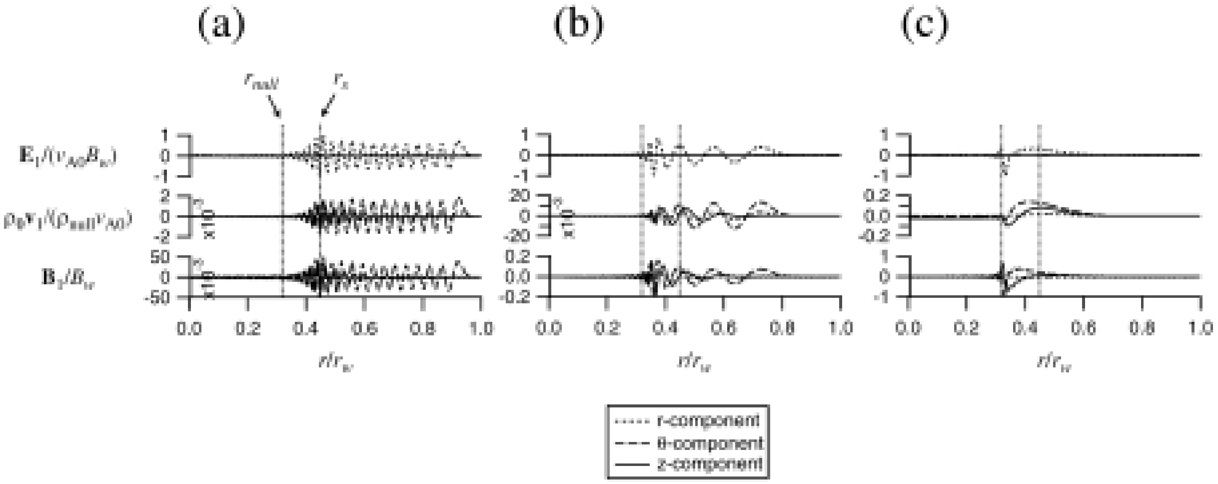

Fig. 3. Radial structure of the perturbed electric field, mass flux density, and magnetic field (the eigenfunctions) of the eigenmodes in the peaked current equilibrium with (a) $r_w k = 0.11$, $\omega/k = 20 v_{A0}$ *; (b)* $r_w k = 0.38$, $\omega/k = 5.4 v_{A0}$ *; (c)* $r_w k = 2.0$, $\omega/k = 1.0 v_{A0}$.

increases the number of nodes in the mass flux density increases, while the electric field keeps the same structure.

In the flat and hollow current equilibria, we see the same trend in radial structure of the wave field as the phase velocity increases.

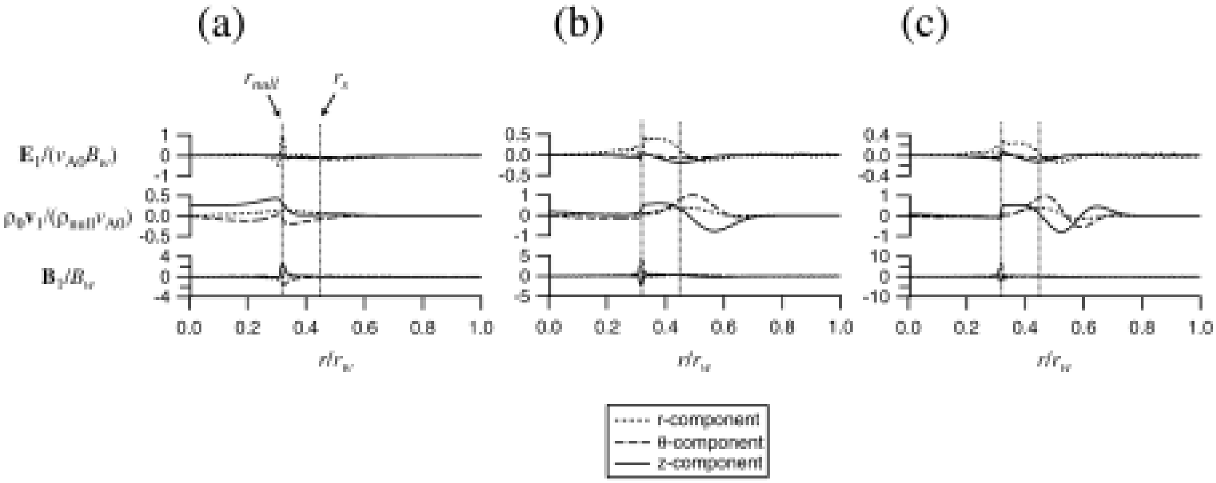

Fig. 4. Radial structure of the perturbed electric field, mass flux density, and magnetic field (the eigenfunctions) of the eigenmodes in the peaked current equilibrium with (a) $r_w k = 4.0$, $\omega/k = 0.51 v_{A0}$; (b) $r_w k = 6.8$, $\omega/k = 0.30 v_{A0}$; (c) $r_w k = 9.2$, $\omega/k = 0.22 v_{A0}$.

5. Discussion

In the experiment [1], the magnetic fluctuation measurement was performed only around the separatrix. The fluctuation in the azimuthal direction, $B_{1\theta}$, was larger than the other components. The phase velocity of the observed wave increases with the radius. The numerical results show that the amplitude of perturbed magnetic field is largest in θ-direction for phase velocities $\omega/k > v_{A0}$. In addition, the mode, which propagates near the magnetic null, has smaller phase velocity than that propagates away from the magnetic null (the mode in Fig. 3b has smaller phase velocity than the modes in Fig. 3a). These trends in the numerical results are consistent with the experimental results.

For a typical FRC plasma $T_i \sim 2T_e$ and $\beta \sim 1$. Thus the thermal velocity of the ion is $v_{th,i} \sim \sqrt{2/3} v_{A0} \sim 0.82 v_{A0}$. Figure 5 shows the modes with phase velocity similar to the ion's thermal velocity in the peaked, flat, and hollow current equilibria. All of these waves have significant B_{1z} between the magnetic null and the separatrix, where the plasma pressure is high. In addition, all those modes have the wave number similar to that of the applied magnetic field in the experiment ($r_w k = 2.1$ for the applied field). Thus this type of mode can accelerate the plasma ions in *z*-direction by one of the collisionless wave-particle interactions, the *transit-time magnetic damping* [5,6]. This leads to a possibility that the ions are accelerated in z-direction by the wave through the transit-time magnetic damping, and the accelerated ions produce the current in z-direction and the toroidal magnetic field as observed in the experiment [2], then the ions are heated by making ion-ion collisions.

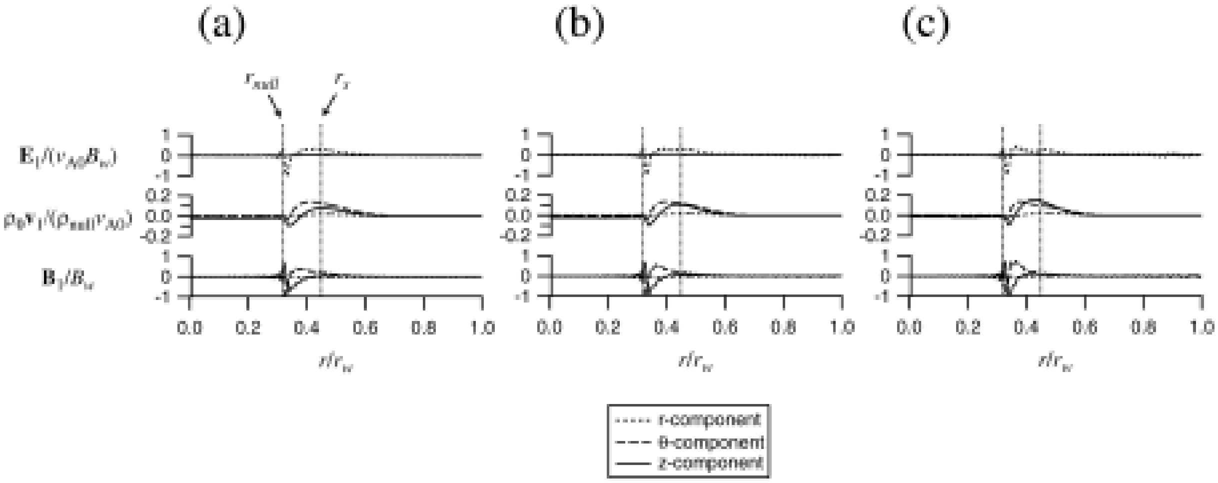

Fig. 5. Radial structure of the perturbed electric field, mass flux density, and magnetic field (the eigenfunctions) of the eigenmodes (a) in the peaked current equilibrium with $r_w k = 2.0$, $\omega/k = 1.0 v_{A0}$ *; (b) in the flat current equilibrium with* $r_w k = 2.2$, $\omega/k = 0.96 v_{A0}$ *; (c) in the hollow current equilibrium with* $r_w k = 2.5$, $\omega/k = 0.84 v_{A0}$.

References

[1] K. Yamanaka *et al.*, Phys. Plasmas **7**, 2755 (2000).

[2] K. Yamanaka Ph.D. thesis, Graduate School of Engineering, Osaka University, Japan, 2000.

[3] Y. Suzuki *et al.*, Phys. Plasmas **7**, 4062 (2000).

[4] L. C. Steinhauer *et al.*, Phys. Fluids B **4**, 645 (2000).

[5] T. H. Stix, *The theory of plasma waves* (McGraw-Hill).

[6] Kenro Miyamoto, *Plasma Physics for Nuclear Fusion* (MIT Press).