Effect of angular momentum on FRC minimum energy states

Preston Geren
Geren Consulting Services

Loren Steinhauer
University of Washington

All stable equilibria are stationary-energy (SE) states of some form. Thus finding the SE states considerably simplifies the search for stable plasma equilibria. This is particularly so in the case of flowing equilibria for which finding any kind of equilibrium is exceedingly complicated. A previous analysis found SE of a two fluid plasma by minimizing the ordered energy W_{MF} (flow + magnetic field energy) subject to constraints on the two integral constants of motion for a two-fluid, the ion helicity K_i and the electron helicity K_e . By expanding the flow and field vectors in a complete basis set of divergence-free vectors, the problem was reduced to a system of algebraic equations. Solution of the equations led to the prediction of SE states that are a two-point spectrum of the basis set, i.e. *double-mode condensates*.[1] Once these states are found the important functional relationship W_{MF} (K_i,K_e) follows, allowing the minimum energy state to be identified.

The foregoing analysis has been extended to include the effect of another constant of motion, the angular momentum, L_z. In the case of an axisymmetric system with suitable boundary conditions (constant magnetic flux boundary, no flow through the boundary, free-slip condition) the angular momentum is an integral constant of motion. Incorporating this additional constraint leads to a more complicated system of equations. Instead of a two-point spectrum of SE states, all *axisymmetric* mode elements are represented except in exceptional cases, although the spectrum tends to concentrate in certain ranges. An important consequence of angular momentum conservation is that the solution is axisymmetric, i.e., tilted equilibrium states are disallowed.

Inspection of the form of the spectrum reveals potential pathological behavior, which must be carefully dealt with in numerical calculations. A method for removing the pathologies is presented and applied. Investigation of the equations indicates that there are two regimes for the SE states, a "no-root" region in which all modes contribute to the conserved quantities, and a "root" region in which the spectrum may collapse to a one- or two-point spectrum. Calculations of the functional form W_{MF} (K_i,K_e,L_z) will be presented for both cases.

[1] L.C. Steinhauer, Phys. Plasmas **9**, October 2002 (in press)

e-mail: pgeren@pop.seanet.com