Multi-linked FRC system with week toroidal fields by using various kinds of energy injections

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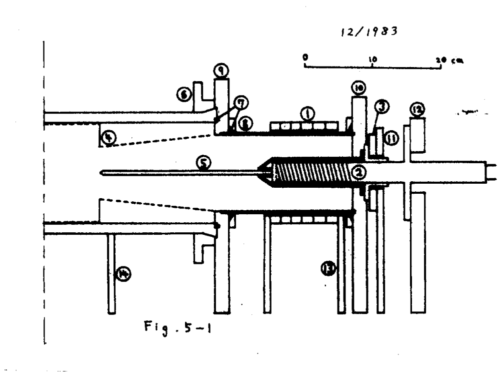

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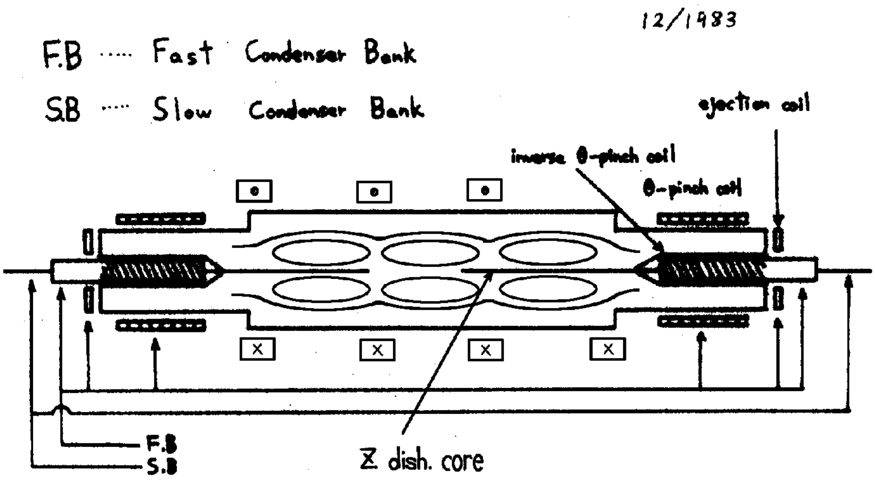

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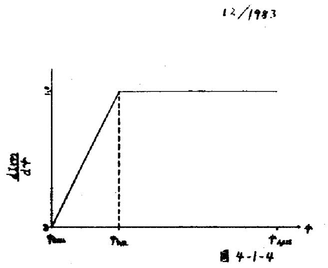

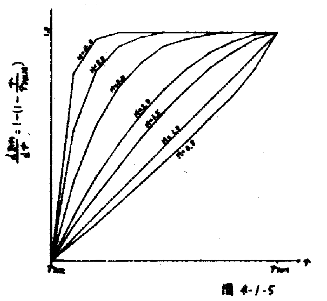

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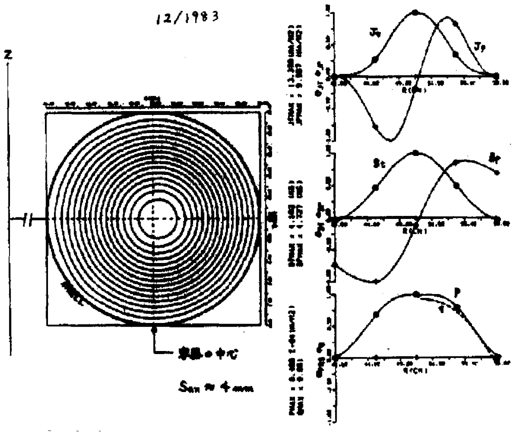

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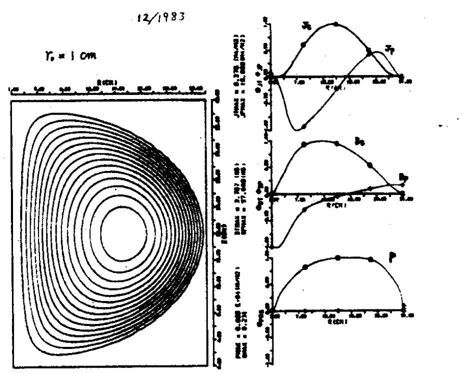

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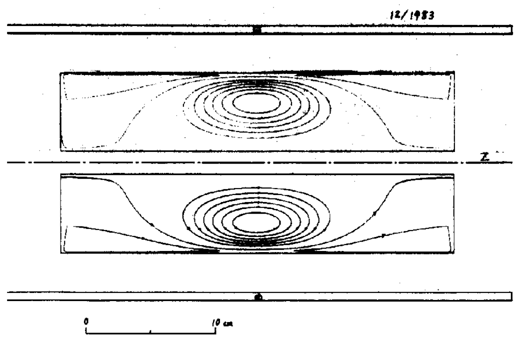

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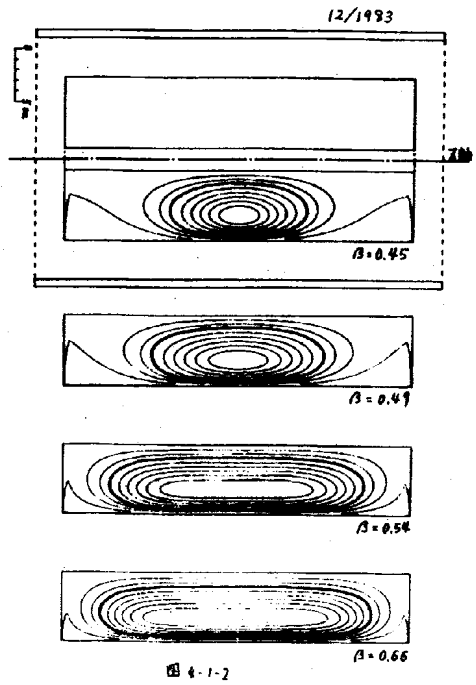

II Magnetic helicity is never the global time invariance ! and a generalized self-organization theory

II.1 Theoretical Thought Analysis

Axiom set of physical laws of Maxwell's equations

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}\,, \tag{1}$$

$$\frac{\partial \mathbf{D}}{\partial t} = -\mathbf{j} + \nabla \times \mathbf{H}\,, \tag{2}$$

$$\nabla \cdot \mathbf{D} = \rho\,, \tag{3}$$

$$\nabla \cdot \mathbf{B} = 0\,. \tag{4}$$

Poynting's energy conservation law for the field energy W_f

$$\frac{\partial W_f}{\partial t} = -\int_V \mathbf{j}\cdot\mathbf{E}\mathrm{d}V - \frac{1}{\mu_0}\oint_S (\mathbf{E}\times\mathbf{B})\cdot\mathrm{d}\mathbf{S}\,, \tag{5}$$

where

$$W_{\rm f} = \int_V (\frac{\mathbf{E}\cdot\mathbf{D}}{2} + \frac{\mathbf{H}\cdot\mathbf{B}}{2}) \,\mathrm{d}V \,, \tag{6}$$

The two physical laws of Eqs.(1) and (4) are rewritten

$$\frac{\partial \mathbf{A}}{\partial t} = -\nabla\phi - \mathbf{E}\,, \tag{7}$$

$$\mathbf{B} = \nabla\times\mathbf{A}\,,. \tag{8}$$

Definition of the magnetic helicity K [J. B. Taylor: Phys. Rev. Lett. **33** (1974) 1139.]:

$$K \equiv \frac{1}{\mu_0}\int_V \mathbf{A}\cdot\mathbf{B}\,\mathrm{d}V\,. \tag{9}$$

Here, we emphasize that even if we include "the external helicity", taking account of the gauge invariance, the following argument is still essentially correct and applicable. "The time change rate of K ", Partial derivative of the definition Eq.(9) with respect to t, and using only two laws of Eqs. (1) and (4) , the vector formulae , and Gauss theorem, "the time change rate of K ",

$$\begin{aligned}\frac{\partial K}{\partial t} &= \frac{1}{\mu_0}\int_V (\frac{\partial \mathbf{A}}{\partial t}\cdot\mathbf{B} + \mathbf{A}\cdot\frac{\partial \mathbf{B}}{\partial t})\,\mathrm{d}V \\ &= -\frac{2}{\mu_0}\int_V \mathbf{B}\cdot\mathbf{E}\mathrm{d}V \\ &\quad + \frac{1}{\mu_0}\oint_S (\mathbf{E}\times\mathbf{A} - \phi\mathbf{B})\cdot\mathrm{d}\mathbf{S}\,. \end{aligned} \tag{10}$$

It should be emphasized here that "the time change rate of K, Eq.(10)" is derived from merely one physical law of Eq.(1), which can never lead to any deterministic time evolutions of $\mathbf{A}$ and $\mathbf{B}$ without another physical law of Eq.(2). The value of $\partial K/\partial t$ is passively and resultantly determined by the mutually independent volume and surface integral terms in Eq.(10). Equation (10) is not "the conservation equation of the helicity K itself", but is merely an equation for "the time change rate of K".

Assuming $(\mathbf{E}\cdot\mathbf{D})/2 \ll (\mathbf{H}\cdot\mathbf{B})/2$ in the plasma confinement experiments, and using the simplified Ohm's law of Eq.(11), "the energy conservation law" and the so-called "helicity conservation law" are derived respectively as

$$\mathrm{Ohm's\ law:}\quad \mathbf{E} + \mathbf{v}\times\mathbf{B} = \eta\mathbf{j}\,. \tag{11}$$

$$\begin{aligned}\frac{\partial W_f}{\partial t} &\cong \frac{\partial W_m}{\partial t} \\ &= -\int_V \{\,\eta\mathbf{j}\cdot\mathbf{j} + (\mathbf{j}\times\mathbf{B})\cdot\mathbf{v}\,\}\,\mathrm{d}V \\ &\quad -\frac{1}{\mu_0}\oint_S (\mathbf{E}\times\mathbf{B})\cdot\mathrm{d}\mathbf{S}\,. \end{aligned} \tag{12}$$

$$\begin{aligned}\frac{\partial K}{\partial t} &= -\frac{2}{\mu_0}\int_V \eta\mathbf{j}\cdot\mathbf{B}\,\mathrm{d}V \\ &\quad + \frac{1}{\mu_0}\oint_S (\mathbf{E}\times\mathbf{A} - \phi\mathbf{B})\cdot\mathrm{d}\mathbf{S}\,. \end{aligned} \tag{13}$$

The misunderstanding on the so-called "helicity conservation law" has been established from the following argument, using "the time change rate equation of K written by Eq.(13)".

a) At first, we consider "the ideal case" where the whole region of plasmas inside the boundary is filled with the ideally conducting plasma and the boundary surface is the ideally conducting wall, i.e., $\eta = 0$ and $\mathbf{E} = 0$ and $\mathbf{B}\cdot\mathrm{d}\mathbf{S} = 0$ at the ideally conducting wall. We then get from Eq.(13) in this "ideal case" that "the time change rate of K" becomes as $\partial K/\partial t = 0$.

b) From this result, we may conclude followings. Since the value of K is constant along the time variable t, the total helicity K is conserved and therefore it must be "the time invariant in the dynamical system in the case of ideal plasmas".

However, the part of (a) declares only that the value of K defined by Eq.(8) does not change along the time variable t in "the special or the trivial case" of $\eta = 0$ plasmas filling fully within the ideally conducting wall.

** A simple thought experiment ** a) Consider that there exists some vacuum field region, i.e., $\eta = \infty$, near the ideally conducting wall, and the other region is still filled with the ideally conducting plasma.

b) We then have to come back to Eq.(10), and we can put $\mathbf{E} = 0$ in the plasma but have to leave $\mathbf{E}$ in the vacuum field region.

c) In this simple case, the value of $\partial K/\partial t$ is passively and resultantly determined by the volume integral of $\mathbf{B}\cdot\mathbf{E}$ in Eq.(10) along the time variable t.

d) The decrement of K in this simple case is by no means "the resistive loss of K", because of no current in the vacuum field region.

e) On the other hand, we definitely know that the changed part of $W_{\rm m}$ transfers to the other type of energy, such as the kinetic energy, inside the ideally conducting wall by Eq.(12).

f) However, the total helicity K can never be conserved in the dynamical system in this simple case. This is because that Eq.(10) is merely an equation for "the time change rate of K", and the helicity K is not the physical quantity but merely represents the topological property of the magnetic field lines at each instant.

g) The simple thought experiment shown above may lead us to a conclusion that the total value of K is never "the time invariant in the dynamical system".

The value of the helicity K has never been conserved in the computer simulations by R. Horiuchi and T. Sato [R. Horiuchi and T. Sato: Phys. Rev. Lett. **55** (1985) 211] and also in all experiments on the reversed field pinch (RFP) by many authors, on the toroidal Z-pinch by Dr. K. Sugisaki at ETL [K. Sugisaki: Jpn. J. Appl. **24** (1985) 328] and on merging two spheromacs into one field reversed configuration (FRC) or one spheromac by Y. Ono, Katsurai et. al. [Y. Ono, M. Yamada, T. Akao, T. Tajima, and R. Matsumoto:

Phys. Rev. Lett. **761** (1996) 3328] . Especially, in the case of the toroidal Z-pinch experiments, the total helicity K increases to finite values from zero initial value within a few tens of μs [K. Sugisaki: Jpn. J. Appl. **24** (1985) 328]. "These experimental results have been demonstrated that the conjecture of the total helicity invariance by Dr. J. B. Taylor is not physically available to real magnetized plasmas." Even if they believe the so-called helicity injection as a technical method, the helicity injection is physically the magnetic energy injection in real experiments. If $\partial W_{\rm m}/\partial t = 0$ is realized by the so-called helicity injection without energy injections, then the process "violates the more important physical law of the energy conservation !"

As is known mathematically, "as a axiom, the starting by the variational principle with the use of variational formulation" $\equiv$ "that by the related or resultant dynamic equations as an axiom set". "The energy principle" is within this physical thought, i.e., it have to lead to the related dynamic equations.

Physically and mathemathically important point is that the axiom set of related dynamic equations give us all kinds of time evolutions of the dynamic system itself, including not only self-organized states of equilibria but also the relaxation processes themselves and all other changing processes.

The relaxation theory by Dr. J. B. Taylor has been neither the variational principle nor the energy principle. Taylor's theory is merely "the variational calculus with global constraint with respect to the value of K" to find the minimum energy solution from the set of solutions having the same value of K.

Without using the concept of the helicity K, we can derive the relaxed state of MHD plasmas as $\nabla \times (\eta \nabla \times \mathbf{B}) = \lambda \mathbf{B}$ including $\nabla \times \mathbf{B} = \lambda_{\rm T} \mathbf{B}$ for a special case of spatially uniform η [Y. Kondoh: J. Phys. Soc. Jpn. **58** (1989) 489. Y. Kondoh: Phys. Rev. E **49** (1994) 5546. Y. Kondoh and J. W. Van Dam: Phys. Rev. E **52**, 1721 (1995).]. Relaxations, the magnetic field generation, and the transformation between the toroidal- and the poloidal magnetic fields are due to the dynamo term of $(\mathbf{j} \times \mathbf{B}) \cdot \mathbf{v}$ in the energy conservation law of Eq.(12), where the velocity $\mathbf{v}$ comes from the Lorentz force and/or the thermal convection of the conducting fluids.

II.2 A Generalized Self-Organization Theory

[Y. Kondoh: Phys. Rev. **E 48** (1993) 2975.]

It should be emphasized here that the generalized self-organization theory with the use of auto-correlations for physical quantities is not fundamentally based on both of the variational principle and the energy principle, and the auto-correlations are never the time invariants.

Quantities with n elements in general dynamic systems of interest shall be expressed as $\mathbf{q}(t, \mathbf{x}) = \{q_1(t, \mathbf{x}), q_2(t, \mathbf{x}), \cdots, q_n(t, \mathbf{x})\}$. Here, t is time, $\mathbf{x}$ denotes m-dimensional space variables, and $\mathbf{q}$ represents a set of physical quantities having n elements. Dissipative nonlinear dynamic system generally described by

$$\frac{\partial q_i}{\partial t} = G_i[\mathbf{q}] \,, \tag{14}$$

where $G_i[\mathbf{q}]$ denotes linear or nonlinear dynamic operators, which may include non-dissipative and/or dissipative terms. In some cases, the operator $G_i[\mathbf{q}]$ may include negative dissipation terms such as energy input terms $q_i(t, \mathbf{x})$ [$i = 1, 2, 3, \ldots$]. After taking the product of $q_i(t, \mathbf{x})$ and both sides of Eq.(14), and integrating the resultant equation over the volume V, "the conservation laws" for the for the dynamic system with $q_i(t, \mathbf{x})$ are derived, as

$$\begin{aligned} &\int_V \{ \frac{\partial}{\partial t} \frac{1}{2} [q_i(t, \mathbf{x}) q_i(t, \mathbf{x})] \} \, {\rm d}V \\ &= \int_V \{ q_i(t, \mathbf{x}) G_i[\mathbf{q}] \} \, {\rm d}V \,. \end{aligned} \tag{15}$$

After defining the axiom set of dynamic equations of Eq.(14), the solving the time evolution of the dynamic system with given boundary conditions belongs to the pure mathematical problems as well as the physical ones. All time evolutions of the dynamic system may go on deterministically by the all Eqs.(14) [$i = 1, 2, 3, \ldots$] that have completive interactions with all other quantities. If we trace all over time evolution of the dynamic system, we may fined that at some phase, one or some dynamic operators work dominantly and others are negligible. Furthermore, we may notice that the dominant operators always interchange with each other as the time goes on. On the way of the time evolution of the dynamic system, we may also find the spatial profile of some quantity $q_i(t, \mathbf{x})$ [$i = 1, 2, 3, \ldots$] becomes unchangeable or a steady state, for which we would call the state " the self-organized state".

We should notice here, however, that there are no reasons to believe the self-organized states for all quantities $q_i(t, \mathbf{x})$ [$i = 1, 2, 3, \ldots$] to appear at the same instant. On the contrary, it is more natural to expect that the self-organized spatial profile would have some shift with each other among the quantities. This is because that if the self-organized steady profiles of all quantities $q_i(t, \mathbf{x})$ [$i = 1, 2, 3, \ldots$] take place at the same instant, then the dynamic system does not evolve in time after that, i.e., this situation contradicts logically to the starting assumption that the system evolves by the axiom set of the dynamic equations !

All of the quantities in the dynamic system go through their "own short rest" on their way, and the dynamic system will show various faces during all over the time evolution. From this standpoint of observation on over all time evolution of the dynamic system, we can identify or define "the self-organized state" for each quantity as "the self-similar state in the phase with the most unchangeable structure".

In order to describe quantitatively those most unchangeable structure for each quantity, we inevitably introduce the auto-correlations as a suitable measure. The definition of "the self-organized state" may be mathematically expressed by using auto-correlations, $q_i(t, \mathbf{x}) q_i(t + \Delta t, \mathbf{x})$, between the time, t, and slightly transferred time, $t + \Delta t$, with a small Δt,

i. e., “t self-organheized state” is defined as follows

$$min \mid \frac{\int q_i(t,\mathbf{x})q_i(t+\Delta t,\mathbf{x})\mathrm{d}V}{\int q_i(t,\mathbf{x})q_i(t,\mathbf{x})\mathrm{d}V} - 1 \mid \text{state.} \tag{16}$$

Taylor expansion $q_i(t+\Delta t,\mathbf{x}) = q_i(t,\mathbf{x}) + [\partial q_i(t,\mathbf{x})/\partial t]\Delta t + (1/2)[\partial^2 q_i(t,\mathbf{x})/\partial t^2](\Delta t)^2 + \cdots$ and the 1st ordr of Δt give us the definition of the self-organized state during the order of the time scale Δt as

$$min \mid \frac{\int q_i(t,\mathbf{x})[\,\partial q_i(t,\mathbf{x})/\partial t\,]\mathrm{d}V}{\int q_i(t,\mathbf{x})q_i(t,\mathbf{x})\mathrm{d}V} \mid \text{ state .} \tag{17}$$

Substituting the dynamic equation, Eq.(14), into Eq.(17), we obtain the equivalent definition for the self-organized state, as

$$min \mid \frac{\int q_i(t,\mathbf{x})G_i[\mathbf{q}]\mathrm{d}V}{\int q_i(t,\mathbf{x})q_i(t,\mathbf{x})\mathrm{d}V} \mid \text{ state .} \tag{18}$$

Eq.(17) shows that realization of the self-similar coherent structures, i. e., “the self-organheized state”, in dynamical systems is the phase of “the minimum change rate of auto-correlations for their instantaneous values”.

Since we have substituted the original dynamic equations into the definition of the self-organized state, “whole properties of the dynamic system is essentially embedded in the process to derive the self-organized state from Eq.(18)”. The mathematical expressions with the use of the variational calculus for Eq.(17) and Eq.(18) are written as follows, defining a functional F with use of a Lagrange multiplier λ_i, as

$$\begin{aligned} F \quad \equiv \quad & \int_V \{\, \frac{\partial}{\partial t}\frac{1}{2}[\, q_i(t,\mathbf{x})q_i(t,\mathbf{x})\,] \\ & + \lambda_i q_i(t,\mathbf{x})q_i(t,\mathbf{x}) \,\}\,\mathrm{d}V \\ = \quad & \int_V \{\, q_i(t,\mathbf{x})G_i[\mathbf{q}] \\ & + \lambda_i q_i(t,\mathbf{x})q_i(t,\mathbf{x}) \,\}\,\mathrm{d}V\,. \end{aligned} \tag{19}$$

$$\delta F \;=\; 0\,, \tag{20}$$

$$\delta^2 F \;>\; 0, \tag{21}$$

where δF and $\delta^2 F$ are respectively the first and the second variations of F “with respect to the variation $\delta\mathbf{q}(\mathbf{x})$ only for the spatial variable $\mathbf{x}$”.

Comparing Eqs.(15) and (19), we can find that the conservation equations for the dynamic system are naturally included in the present formulation of the generalized self-organization theory. We should remind here that the global auto-correlation $\int q_i(t,\mathbf{x})q_i(t,\mathbf{x})\mathrm{d}V$ is never the time invariant but strictly the global constraint.

The implicit assumption in this theory is that the dynamical system evolves all possible area in state phases.

II.3 Application to Plasmas

We apply here the generalized self-organization theory shown above to fusion plasmas. According to the general type of the dynamic equations, Eq.(14), we rewrite Eqs.(1) and (2) as follows

$$\frac{\partial \mathbf{B}}{\partial t} \;=\; -\,\nabla\times\mathbf{E}\,. \tag{22}$$

$$\varepsilon_0\frac{\partial \mathbf{E}}{\partial t} \;=\; \nabla\times\mathbf{H} \,-\, \mathbf{j}\,. \tag{23}$$

Following three more physical laws, i.e., the conservation laws of the mass, Eq.(24), the momentum, Eq.(25), and the generalized Ohm’s law, Eq.(26).

$$\frac{\partial \rho_\mathrm{m}}{\partial t} \;=\; -\,\nabla\cdot(\,\rho_\mathrm{m}\,\mathbf{v}\,)\,. \tag{24}$$

$$\begin{aligned} \rho_\mathrm{m}\,\frac{\partial \mathbf{v}}{\partial t} \quad = \quad & -\,\rho_\mathrm{m}\,(\mathbf{v}\cdot\nabla)\mathbf{v} + [\,\rho_\mathrm{e}\mathbf{E} \,+\, \mathbf{j}\times\mathbf{B} \\ & -\nabla(\,P_\mathrm{e} \,+\, P_\mathrm{i}\,)\,]\,. \end{aligned} \tag{25}$$

$$\begin{aligned} \frac{\partial \mathbf{j}}{\partial t} \quad = \quad & \frac{e^2 n_\mathrm{e}}{m_\mathrm{e}}\,\{\mathbf{E} \,+\, \mathbf{v}\times\mathbf{B} - \eta_\mathrm{ei}\,\mathbf{j} - \frac{1}{e n_\mathrm{e}}(\mathbf{j}\times\mathbf{B}) \\ & + \frac{1}{e n_\mathrm{e}}\,[\,\nabla P_\mathrm{e} - (m_\mathrm{e}/m_\mathrm{i})Z_\mathrm{i}\nabla P_\mathrm{i}\,]\,\}. \end{aligned} \tag{26}$$

These three equations come from the Boltzmann kinetic equations for electrons and ions.

We start with the axiom set of seven physical laws of Eqs.(1),(2),(3), (4), (24), (25) and (26). Poynting’s energy conservation law concerning with the field energy W_f is

$$\frac{\partial W_f}{\partial t} \;=\; -\int_V \mathbf{j}\cdot\mathbf{E}\mathrm{d}V \,-\, \frac{1}{\mu_0}\oint_S(\mathbf{E}\times\mathbf{B})\cdot\mathrm{d}\mathbf{S}\,. \tag{27}$$

The conservation law of the kinetic energy $W_\mathrm{k} = \int_V (\rho_\mathrm{m}/2)\mathbf{v}\cdot\mathbf{v}\,\mathrm{d}V$ is

$$\begin{aligned} \frac{\partial}{\partial t}W_\mathrm{k} \quad = \quad & \int_V \{-\,\frac{1}{2}\mathbf{v}\cdot\mathbf{v}\,\nabla\cdot(\,\rho_\mathrm{m}\,\mathbf{v}\,) \\ & -\,\rho_\mathrm{m}\,\mathbf{v}\cdot[(\mathbf{v}\cdot\nabla)\mathbf{v}] \\ & +[\,\rho_\mathrm{e}\,\mathbf{E}\cdot\mathbf{v} \,+\, (\mathbf{j}\times\mathbf{B})\cdot\mathbf{v} \\ & -\,\rho_\mathrm{m}\,\mathbf{v}\cdot\nabla(\,P_\mathrm{e} \,+\, P_\mathrm{i}\,)\,]\,\}\,\mathrm{d}V\,. \end{aligned} \tag{28}$$

A conservation law on the current by defining $W_\mathrm{c} = \int_V (1/2)\mathbf{j}\cdot\mathbf{j}\,\mathrm{d}V$ is obtained as

$$\begin{aligned} \frac{\partial}{\partial t}W_\mathrm{c} \quad = \quad & \int_V \frac{e^2 n_\mathrm{e}}{m_\mathrm{e}}\,\{\,\mathbf{j}\cdot\mathbf{E} \,-\, (\mathbf{j}\times\mathbf{B})\cdot\mathbf{v} \\ & -\eta_\mathrm{ei}\,\mathbf{j}\cdot\mathbf{j} + \frac{1}{e n_\mathrm{e}}\,\mathbf{j}\cdot[\,\nabla P_\mathrm{e} \\ & -(m_\mathrm{e}/m_\mathrm{i})Z_\mathrm{i}\nabla P_\mathrm{i}\,]\,\}\,\mathrm{d}V\,. \end{aligned} \tag{29}$$

According to Eq.(19), we obtain the functional for the field energy $F_\mathbf{f}$, for the kinetic energy $F_\mathbf{k}$, and for the current $F_\mathbf{c}$, respectively, as

$$\begin{aligned} F_\mathbf{f} \quad = \quad & \int_V \{\,-\,\mathbf{j}\cdot\mathbf{E} \,+\, \lambda_\mathrm{f}\,(\,\frac{\varepsilon_0\mathbf{E}\cdot\mathbf{E}}{2} \\ & + \frac{\mathbf{B}\cdot\mathbf{B}}{2\mu_0}\,)\,\}\mathrm{d}V \,-\, \frac{1}{\mu_0}\oint_S(\mathbf{E}\times\mathbf{B})\cdot\mathrm{d}\mathbf{S}, \end{aligned} \tag{30}$$

$$
\begin{aligned}
F_{\mathbf{k}} \;=\; & \int_V \{ -\frac{1}{2}\mathbf{v}\cdot\mathbf{v}\,\nabla\cdot(\,\rho_{\mathrm{m}}\,\mathbf{v}\,) \\
& -\rho_{\mathrm{m}}\,\mathbf{v}\cdot[(\mathbf{v}\cdot\nabla)\mathbf{v}] + \;[\,\rho_{\mathrm{e}}\mathbf{E}\cdot\mathbf{v} \\
& +(\mathbf{j}\times\mathbf{B})\cdot\mathbf{v} - \;\rho_{\mathrm{m}}\,\mathbf{v}\cdot\nabla(\,P_{\mathrm{e}}\,+\,P_{\mathrm{i}}\,)\,] \\
& +\lambda_{\mathrm{v}}\,\frac{\rho_{\mathrm{m}}}{2}\mathbf{v}\cdot\mathbf{v}\,\}\;\mathrm{d}V\;, \qquad (31)
\end{aligned}
$$

$$
\begin{aligned}
F_{\mathrm{c}} \;=\; & \int_V \frac{e^2 n_{\mathrm{e}}}{m_{\mathrm{e}}}\{[\mathbf{j}\cdot\mathbf{E}\,-(\mathbf{j}\times\mathbf{B})\cdot\mathbf{v} - \eta_{\mathrm{ei}}\mathbf{j}\cdot\mathbf{j}\,] \\
& +\frac{e}{m_{\mathrm{e}}}\,\mathbf{j}\cdot[\,\nabla P_{\mathrm{e}} - (m_{\mathrm{e}}/m_{\mathrm{i}})Z_{\mathrm{i}}\nabla P_{\mathrm{i}}\,] \\
& +\lambda_{\mathrm{c}}\,\mathbf{j}\cdot\mathbf{j}\,\}\;\mathrm{d}V\;. \qquad (32)
\end{aligned}
$$

In general, we take variations with respect to $\delta\mathbf{E}$, $\delta\mathbf{B}$, $\delta\mathbf{v}$, $\delta\mathbf{j}$, $\delta\rho_{\mathrm{m}}$, $\delta\rho_{\mathrm{e}}$, δP_{e}, δP_{i}, δn_{e}, δn_{i}, and $\delta\eta_{\mathrm{ei}}$. From the Euler-Lagrange equations for the solutions of Eq.(20), we will get new various equilibrium configurations of the self-organized states with the plasma flow, the shear flow, the space charge, the space potential, the deviation between the ion and the electron density profiles, the resistivity profile, and so on, depending on the boundary conditions and the external input sources such as the various energy injections and the particle beams. The resultant equilibrium configurations are far beyond the conventional MHD equilibrium ones by the Grad-Shafranov equation based on the equation of $\mathbf{j}\times\mathbf{B} = \nabla p$.

In order to realize the steady state of the confinement system of plasmas, we can extend conventional methods of plasma current drives, by using the three conservation laws of Eqs.(27),(28) and (29),i.e., using energy injections of various types of energies, such as magnetic energies, electromagnetic wave energies, internal energies of plasmoids by plasma guns, which induce the thermal plasma flow velocity, various particle beam energies, and so on.

III Concluding Remarks

In Section I, we have shown our experimental and numeridal works on the compact toroidal plasmas (CTP) in Gunma Univ., which was investigated around 1983. The experimental work had attempted to produce the CTP and to marge two CTP, such as the extreamly low aspect ratio RFP, the high β spheromack, the Spherical Tokamak (ST), the FRC and/or the FRC with week toroidal magnetic fields, all of which have no axial center conducter with the use of the axisal discharge through the long gap between the two cylindrical electrodes. We have also proposed the Mutli-linked FRC system with week toroidal fields in order to realize high β CTP with long confinement times of the energy and the configurations. In Section II-I, we have demonstrated mathematically that the magnetic helicity K is never the global time invariant We have shown that the so-called "helicity conservation law" is not the conservation law, but is merely an equation of "the time change rate of K which is passively and resultantly determined by the muturally independent volume and surface integral terms in Eq.(10) and/or Eq.(13). We have clarified that the relaxation theory by Dr. J. B. Taylor has been neither the variational principle nor the energy principle, and have discussed that the so-called Taylor state $\nabla\times\mathbf{B} = \lambda_{\mathrm{T}}\mathbf{B}$ with spatially constant λ_{T} and initial K has never been realized in experimental plasmas and in computer simulations. In Section II-I, we have presented a formulation of the advanced generalized self-organization theory that can be applicable any dynamical systems and includes naturally both of the dynamical properties and the conservation equations of the system. We have also shown the application of the theory to the fusion plasma with the use of the axiom set of seven physical laws, and have pointed that the Euler-Lagrange equations for every variations of the quantities lead to their own equilibirium configurations certain time deviation among the quantities, like as the experimental plasmas. The resultant equilibrium configurations are far beyond the conventional MHD equilibrium ones by the Grad-Shafranov equation. In order to realize steady states, we have proposed various types of energy injections , basing on the three conservation laws of Eqs.(27),(28) and (29).