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Eigenmode Analysis of Low Frequency Waves in Field-Reversed Configurations

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I. Introduction

Low frequency waves have been used for plasma heating. Recently a heating experiment of a Field-Reversed Configuration (FRC) plasma has been performed by Yamanaka *et al*. [1]. In this experiment, low frequency (compared with the ion's cyclotron frequency in the external magnetic field) oscillating magnetic field was applied to the FRC plasma. The applied field was homogeneous in the azimuthal direction. As a result, a fluctuation of the magnetic field propagating in the direction parallel to the equilibrium magnetic field of the FRC plasma was observed. In addition, increase of the plasma energy was observed. It was found from comparison of the total temperature and the ion temperature that the increase in the plasma energy was mostly due to the increase in the ion temperature. This implies that the applied magnetic field excited low frequency waves and the energy was absorbed by the ions. In this study eigenmodes of low frequency waves in a FRC plasma is analyzed to understand the heating mechanism using the single-fluid MHD model.

II. Eigenmode analysis

To investigate the low frequency waves propagating through FRC plasmas, the single-fluid MHD equations are used:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho\, \mathbf{v}) = 0 \tag{1}$$

$$\rho\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right)\mathbf{v} = \mathbf{j} \times \mathbf{B} - \nabla p \tag{2}$$

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \frac{1}{en}\left(\mathbf{j} \times \mathbf{B}\right) \tag{3}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{4}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} \tag{5}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{6}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) p \rho^{-\gamma} = 0 \tag{7}$$

Linearizing these equations and assuming that the FRC plasma has no flow in its equilibrium state ($\mathbf{v}_0 = 0$), we have

$$\omega^2 \rho_0 \mathbf{v}_1 - \mathbf{j}_0 \times (\nabla \times \mathbf{E}_1) - \frac{1}{\mu_0} (\nabla \times \nabla \times \mathbf{E}_1) \times \mathbf{B}_0 + \nabla [\mathbf{v}_1 \cdot \nabla p_0 + \gamma \, p_0 \nabla \cdot \mathbf{v}_1] = 0 \tag{8}$$

$$i\omega e n_0 (\mathbf{E}_1 + \mathbf{v}_1 \times \mathbf{B}_0) + e\mathbf{E}_0 \nabla \cdot (n_0 \mathbf{v}_1) - \mathbf{j}_0 \times (\nabla \times \mathbf{E}_1) - \frac{1}{\mu_0} (\nabla \times \nabla \times \mathbf{E}_1) \times \mathbf{B}_0 = 0 \tag{9}$$

where the subscripts '0' and '1' indicate the equilibrium and perturbed quantities, and the perturbed quantities are assumed to have following form in the cylindrical coordinate:

$$f_1(r, \theta, z, t) = \tilde{f}_1(r) \exp[i(m\theta + kz - \omega t)] \tag{10}$$

Equations (8) and (9) constitute an eigenvalue problem. We look for eigenvalues k and corresponding eigenfunctions $i\tilde{v}_{1r}(r), \tilde{v}_{1\theta}(r), \tilde{v}_{1z}(r), \tilde{E}_{1r}(r), i\tilde{E}_{1\theta}(r), i\tilde{E}_{1z}(r)$ for a given ω. The boundary conditions are as follows:

$\mathbf{v}_1 = 0$, $E_{1r} \neq 0, E_{1\theta} = E_{1z} = 0$ at the wall ($r = r_w$)

$v_{1z} \neq 0, v_{1r} = v_{1\theta} = 0$, $E_{1z} \neq 0, E_{1r} = E_{1\theta} = 0$ at the geometric axis ($r = 0$)

The problem is solved in the following way. We approximate the eigenfunctions in terms of a finite series of basis functions $\phi_n(r)$ and a function which satisfies the boundary conditions. For example, iv_{1r} is expressed as follows:

$$iv_{1r}(r) = F_{vr}(r) \sum_{n=1}^{N} C_n^{(vr)} \phi_n(r) \tag{11}$$

where $F_{vr}(r)$ is the function satisfying the boundary condition for iv_{1r} and $C_n^{(vr)}$ are the expansion coefficients. In deriving the equations for the expansion coefficients, we use the *Galerkin method*. Namely, substituting Eq. (11) and expressions for the other

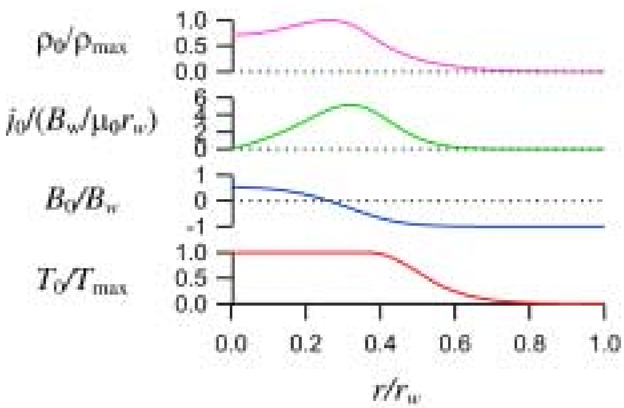

Fig. 1. Profiles of the FRC equilibrium mass density, current density, magnetic field, and temperature. The equilibrium has $K = 0.6$ and $r_s = 0.37$.

variables into Eqs. (8) and (9), multiplying them by $\phi_l(r)$ for $l = 1, \ldots, N$, and integrating them over $0 \le r \le r_w$, we obtain a system of $6N$ equations for $6N$ expansion coefficients. Here r_w is the wall radius.

One-dimensional FRC equilibrium model, which is homogeneous in θ and z-direction, is used in this study. As the equilibrium magnetic field the rigid-rotor profile [2] is used:

$$B_0(r) = B_w \tanh\left[K\left(2r^2/r_s^2 - 1\right)\right] \tag{12}$$

where r_s and B_w are the separatrix radius and the magnetic field at the wall, and K is a parameter. The equilibrium pressure profile is calculated from the pressure balance:

$$p_0(r) = p_{null}\mathrm{sech}^2\left[K\left(2r^2/r_s^2 - 1\right)\right] \tag{13}$$

where p_{null} is the pressure at the field-null point. As the temperature the following profile is used:

$$T_0(r) = \begin{cases} T_{\max} & \text{for } 0 \le r \le r_s \\ T_{\max}\exp\left[-(r-r_s)^2/(2r_e)^2\right] & \text{for } r_s \le r \le r_s + 2r_e \\ (T_{\max}/e)\sqrt{p_0(r)/p_0(r_s+2r_e)} & \text{for } r_s + 2r_e \le r \le r_w \end{cases} \tag{14}$$

where $T_{\max}$ is the maximum temperature. Figure 1 shows the equilibrium profiles.

III. Results

The eigenvalue problem was solved for the azimuthal mode number $m = 0$. The current applied in the experiment has the same mode number. Figure 2 shows the dispersion relation for the low frequency waves propagating through the FRC plasma in z-direction. The frequency is normalized to $\omega_{ci0} \equiv eB_w/m_i$, which is the ion cyclotron frequency in the external magnetic field and the wave number is normalized to $1/r_w$. The broken line corresponds to 80 kHz which is the frequency of the applied field in the

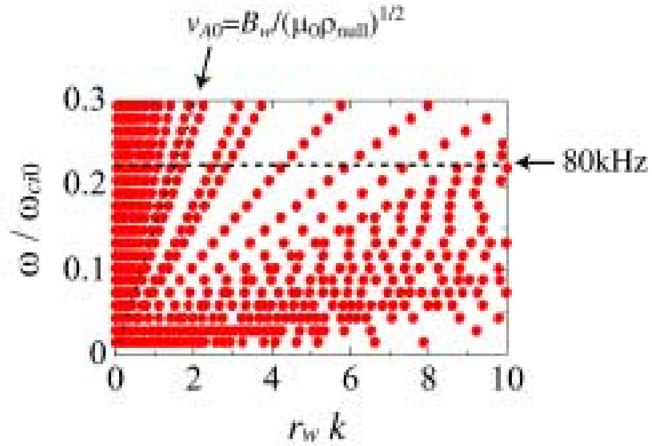

Fig. 2. Dispersion relation of the low frequency waves.

experiment [1]. The dotted line shows the Alfvén velocity based on B_w and ρ_{null} which is the mass density at the field-null point. We see from Fig. 2 that the eigenmodes are dense for $v_{ph} \geq v_{A0}$ and sparse for $v_{ph} \leq v_{A0}$. For the dense region, the density of eigenmodes increases with increase of the number of basis functions N. This shows that there is a continuous spectrum for $v_{ph} \geq v_{A0}$.

Figure 3, 4, and 5 show the radial structure of the perturbed electric field, mass flow, and magnetic field for the continuous spectrum along the broken line (80kHz). For the smaller wavenumbers $r_w k$ (larger phase velocities), the perturbed quantities have their amplitude only outside the separatrix as shown in Fig. 3(a). As $r_w k$ increases (the phase velocity decreases), the amplitudes move to the inner region (Figs. 3(b) and (c)). In other words, modes with larger phase velocity propagate in the outer region in r. In addition, the θ-component of the magnetic field is greater than the other components. This may explain the observed $B_{1\theta}$ in the experiment for which the phase velocity varies with r. For the wavenumbers $r_w k \geq 0.11$, modes such as shown in Fig. 4(a) appear. For these modes the perturbed quantities have their amplitudes only in $r \leq r_{null}$, where r_{null} is the radial position of the field-null point. For

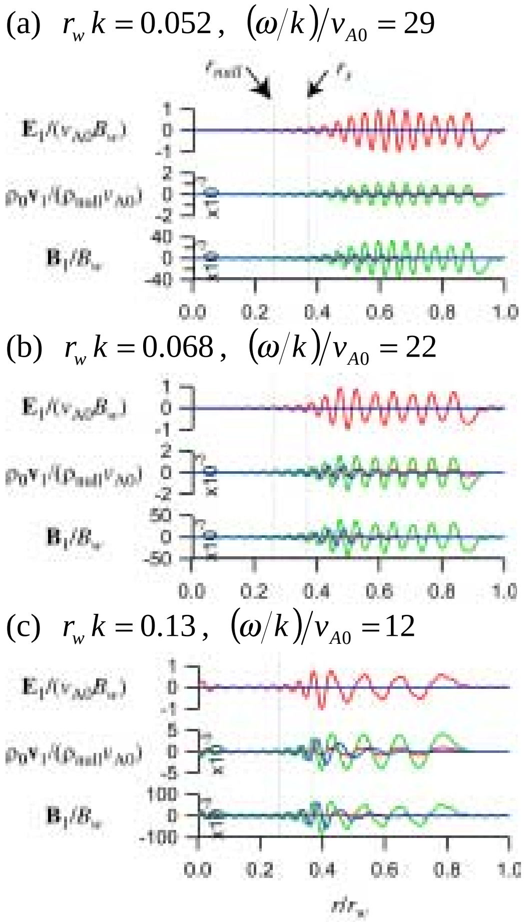

Fig. 3. The radial structure of the perturbed quantities of the eigenmodes for various eigenvalues $r_w k$. The red, green, and blue lines show the r, θ, and z-components, respectively.

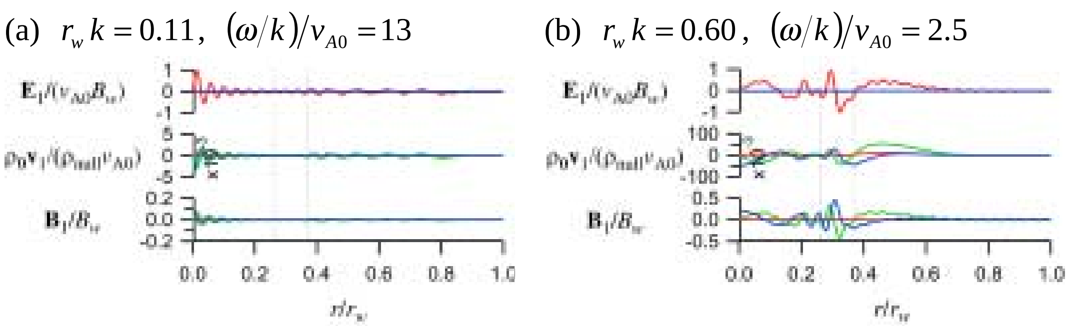

Fig. 4. The radial structure of the perturbed quantities of the eigenmodes.

(a) $r_w k = 4.3$, $(\omega/k)/v_{A0} = 0.35$ (b) $r_w k = 9.5$, $(\omega/k)/v_{A0} = 0.16$

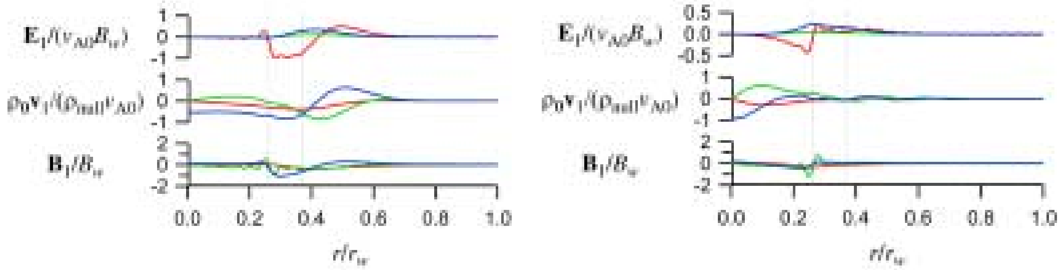

Fig. 5. The radial structure of the perturbed quantities of the eigenmodes.

larger wavenumbers modes such as shown in Fig. 4(b) appear. For these modes, the perturbed quantities have their amplitudes across the field-null point. When the wavenumber $r_w k$ exceeds about 2 the characteristics of eigenmodes change. The radial structure of the amplitude becomes more global as shown in Fig. 5. In addition, significant E_{1z} appears. From Eq. (9) we have

$$E_{1z} = -\frac{1}{en_0} j_{0\theta} B_{1r} \qquad (15)$$

Thus these modes can be strongly affected by the equilibrium current density profile. In a high-β FRC plasma ($\beta \sim 1$) the ion's thermal speed $v_{Ti} \sim v_{A0}$. The phase velocities of these modes are rather small compared with the ion's thermal speed. The current profile of FRC plasmas in experiments may be different from the one used in this study [3]. Thus there is a possibility that these modes have a phase velocity nearly equal to the ion's thermal velocity and strongly damped to heat the ions. Analyses for other FRC equilibria which reflect the experiments [4] are needed.

IV. Summary

In this study the eigenmodes of low frequency waves in a FRC plasma was obtained for the first time. It was found from the dispersion relation that there is a continuous spectrum and discrete spectra. The modes corresponding to the continuous spectrum may explain observed $B_{1\theta}$ in the experiment. Analyses for the equilibria that reflect the experiments are issues in the future.

References

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